A Design Manual for Micromachines using Casimir Forces: Preliminary Considerations

G. Jordan Maclay

Quantum Fields LLC
20876 Wildflower Lane, Richland Center, WI 53581
PH: 608-647-6769; FAX: 608-647-5494; e-mail: jordanmaclay@quantumfields.com

Abstract. General properties of the Casimir force are reviewed, with particular attention to the effects of geometry. Using the conservation of energy, the forces in several simple idealized structures are derived, including the lateral forces on partially overlapping parallel plates, the force in a Casimir "comb" drive with several interleaved surfaces, and the average force when a plate is inserted into a rectangular cavity. The properties of rectangular cavities with a moveable piston are discussed, and illustrated with numerical computations. An oscillating structure is proposed.

INTRODUCTION

Casimir predicted the existence of an attractive force between two infinite parallel uncharged perfectly conducting plates in vacuum almost 50 years ago (Casimir, 1948). This force arises because of the boundary conditions that the quantized source-free electromagnetic field must meet at the metal surfaces as discussed in an extensive review (Plunian et al, 1986). The prediction came very shortly after (Bethe,1948) and (Welton,1948) explained the Lamb shift in the hydrogen atom as due to interaction of the electron with the quantized vacuum electromagnetic field. The Casimir force was a startling and unexpected mesoscopic phenomena arising from the presence of surfaces in the quantized vacuum field. The force was predicted to vary as the inverse fourth power of the separation between the plates. At a separation of 100 nm the predicted force/area was equivalent to about 10^{-4} atm; at 10 nm it was about one atmosphere. The Casimir force has also been computed using the alternative language of source theory and radiative reaction, without explicit reference to vacuum fluctuations by (Schwinger, 1978) and (Milonni, 1994). Since Sparnaay's first attempts in 1959, various measurements have been made on dielectrics (Sparnaay, 1989) that have generally verified the theory of Casimir forces as developed for dielectrics by (Lifshitz, 1956), but not until quite recently was the existence of this attractive Casimir force between metallized surfaces verified, in two separate experiments. (Lamoroux, 1997) used a torsion pendulum with an electromechanical feedback system to measure the Casimir force between a metallized spherical lens and a flat plate to an accuracy of about 5-10% for separations of about 0.6 micrometer to 6 micrometer. (Mohideen, 1998) used an Atomic Force Microscope to measure the force between a metallized optical flat and a metallized ball mounted on the AFM cantilever, obtaining a precision of about 1% for separations of 0.1 to 0.9 micrometers. As measured by the AFM, the forces on an effective area of approximately 10 micrometers^2 were in the picoNewton range. It should be mentioned that if the distance between the surfaces is much less than the wavelength corresponding to the plasma frequency of the metal, then the force becomes the unretarded van der Waal's force.

With the advent of improved methods of making micron and submicron structures, such as microfabrication technology, it has become possible to explore forces arising from quantum fluctuations in greater detail. For example, the cantilever used by (Mohideen, 1998) is a silicon micromachined device, often called a MEMS device (Micro-electromechanical System). The small separation between neighboring surfaces in various MEMS devices means it is possible that the presence of Casimir forces may result in adjacent surfaces being attracted to or sticking to each other (Serry, 1998). Using micromachining methods, a variety of MEMS structures in which vacuum stresses are present can be fabricated. A harmonic oscillator with a Casimir interaction has been modeled but not yet built (Serry, 1995). Some structures, including cavities, can be built to investigate Casimir forces especially using AFM methods (Maclay, 1998).

TABLE 1. Casimir Forces at zero Kelvin for Different Perfectly Conductive Geometries
(only cut-off independent, geometry dependent terms are included)

Parallel Plate (area = L x L, spacing=a)	**Cube** (side=a)	**Sphere** (radius=r)
$-0.0411\nabla c L^2/ a^4$	$+0.0305\nabla c/ a^2$	$+0.0462\nabla c/ r^2$

In this paper, we give some preliminary considerations about possible structures and approaches that one might employ in designing MEMS devices that utilize Casimir forces, particularly with parallel plates and rectangular cavities. The analysis includes the role of the forces and the energy balance, assuming the ambient temperature is absolute zero and the ambient pressure is zero.

GENERAL COMMENTS ON VACUUM ENERGY AND CASIMIR FORCES FOR DIFFERENT GEOMETRICAL STRUCTURES

Casimir forces have been computed for only a few common geometries (See Table 1). The most common one is the infinite, parallel plate geometry that produces an attractive force between the plates. If we examine the infinite parallel plate geometry more fully, and imagine placing perfectly conductive, metal surfaces normal to the parallel plates in order to enclose the volume between the plates, then there would be outward forces on these additional four infinitely long, narrow, surfaces (Brown, 1969). For a conducting spherical shell (Milton, 1978) and (Boyer, 1968) have predicted outward or repulsive Casimir force. (Lukosz, 1971) has predicted repulsive forces for a conducting hollow cube. The vacuum stress on two intersecting planes is attractive. For conductive rectangular cavities with square cross section (*1 x 1 x c*) the Casimir energy and the forces on the cavity walls have been computed by (Ambjorn, 1983), (Haycan, 1993), and (Mostepanenko, 1997) with the result that the forces can be inward or outward depending on the specific dimensions. (Ambjorn , 1983) has computed the constant energy contours for a1 x a2 x a3 geometry for the region *a2, a3* > 1. (Maclay, 1999) has computed the energy and force for rectangular cavities of arbitrary dimension.

For the parallel plate, sphere, cube, and rectangular cavity expressions for the stress-energy tensor $T^{\mu\nu}$ have been derived, giving separate expression for the stresses T^{ij} and energy density T^{00}. It is very important to note that for all geometries for which energy densities and forces have been computed, the expression for the force can also be obtained from the expression for the energy density by the principal of virtual work, which is based on the conservation of energy. By this principal, the force F_x in the *x*-direction equals -¶*E*/¶*x*, where *E* is the total energy in the volume. Thus in microdevices utilizing Casimir forces, we expect to conserve the total energy, which includes the mechanical energy and the field energy.

Geometries with curved surfaces or intersecting planes as discussed by (Deutsch, 1979) and (Balian, 1978) present special problems with respect to vacuum energy which we mention briefly. For gently curved conductors, (Deutsch, 1979) has shown that the stress-energy tensor is approximately proportional to the sum of the reciprocals of the two principal radii of curvature and varies inversely as the cube of the distance to the surface. It follows that the total vacuum energy in any compact region that actually contains part of the curved conducting surface is infinite.

Infinities of this type represent a breakdown of the perfect-conductor approximation. The ether cannot store an infinite amount of energy (whether positive or negative) in a compact region, nor can the conductor support the infinite stresses (DeWitt, 1989). The perfect-conductor boundary conditions are pathological, and lead to an infinite physically observable gravitational field (Deutsch, 1979). For a real metal the electrons are unable to follow an applied electromagnetic field at frequencies above the plasma frequency of the metal. Thus for frequencies above the plasma frequency, the zero-point electromagnetic field does not meet the boundary conditions for an ideal conductor. Also the corners of a real conductive surface are not infinitely sharp, but rounded, eliminating divergences.

SOME SIMPLE DYNAMICAL STRUCTURES

We consider the forces and energy balance in several simple structures with: 1. moving parallel plates; 2. moving plates inserted in rectangular cavities, and 3. pistons moving in rectangular cavities.

Structures using Parallel Plates

Consider two conducting, square, parallel plates, a distance L on each side, that are a distance a apart, with $a<<L$. If we allow the upper plate to approach the lower (fixed) plate quasistatically, then the attractive force $F_{pp}(a)= -KL^2/a^4$ does positive mechanical work during this reversible thermodynamic transformation. We are neglecting edge effects by assuming that the force is proportional to the area. During the transformation, the vacuum energy $E_p(a) = -KL^2/3a^3$ between the plates will be reduced, conserving the total energy in the system. If the separation decreases from a_i to a_f, then the energy balance is

$$E_p(a_f) - E_p(a_i) = -\int_{ai}^{af} F_{pp}(a)da \ . \tag{1}$$

If we then separate the plates quasistatically, letting a increase from a_f to a_i, we do work on the system to restore it to its initial state. Over the entire cycle no net work is done, and there is no net change in the vacuum energy.

Consider an alternative cycle that numerous investigators have proposed in order to extract energy from the vacuum fluctuations. After the plates have reached the point of minimum separation, slide the upper plate laterally until it no longer is opposite the lower plate, then raise the upper plate to its original height, and slide it laterally over the lower fixed plate. Then we allow the plates to come together as before, extracting energy from the vacuum fluctuations and doing mechanical work. If no energy were expended in moving the plate laterally, then this cycle would indeed result in net positive work equal to the energy extracted from the vacuum. Although no one has yet computed in detail the lateral forces between offset parallel plates, it is probable that such forces are not zero, and that no net extraction of energy occurs for this cycle. We do know that the vacuum energy is not altered by a single infinite conducting plate (see DeWitt(1989)). If we neglect Casimir energy "fringing fields," and assume that the energy density differs from the free field density only in the region in which the square (LxL) plates overlap a distance x, where $0<x<L$ (see Fig. 1a), then we can compute the lateral force F_{L2} between the two plates using the conservation of energy (principal of virtual work):

$$F_{L2}(x) = -d\,[-KLx/a^3]/dx = KL/a^3 \ . \tag{2}$$

When one includes the work done by this constant attractive lateral force, which tends to increase x or pull the plates so they have the maximum amount of overlap, then there is not net change in total energy (mechanical plus field) as x goes from 0 to L. The normal Casimir force between these plates when they are directly opposite, with complete overlapping ($x = L$), is L/a times larger than the constant lateral force given by Eq. (2).

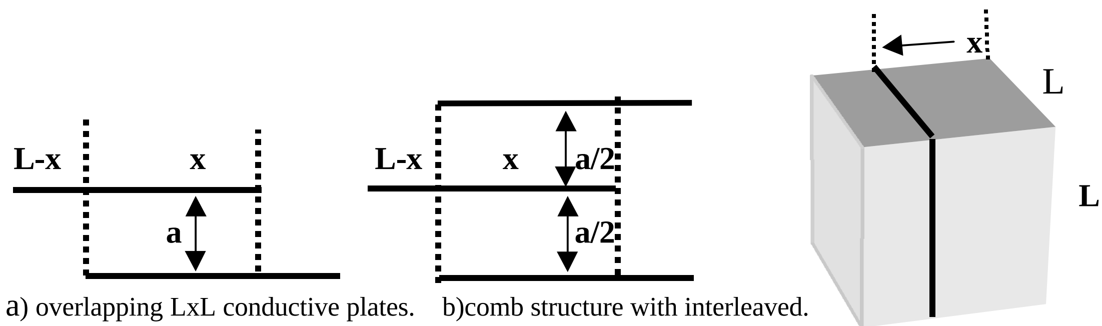
Consider the case of two fixed, square (LxL), parallel plates separated by a distance a, with a third moveable plate that slides in between the two parallel plates, separated by a distance a/2 from each plate (Fig. 1b). If we neglect vacuum energy "fringing fields", as before, that the vacuum energy is different from zero only in regions between directly opposing plates, and we can compute the lateral force on the moveable plate in the middle as minus the derivative of the vacuum energy. The energy, as a function of the overlap x of the fixed and moveable plate, is

$$E(x) = -2KLx/(a/2)^3 - KL(L-x)/a^3 \ , \tag{3}$$

which yields a lateral force $F_{L3} = -dE(x)/dx$ equal to

$$F_{L3} = -15\ KL/a^3 \ . \tag{4}$$

This force is $15a/L$ times the normal Casimir force between the plates separated by a distance a. For a device with a = 0.1 micrometer, L = 1 mm, the lateral force would be an easily measurable 31 nanoNewtons. This structure is analogous to the electrostatic comb drive that is used extensively in MEMS (microelectromechanical systems) devices. One key operational difference between the Casimir and electrostatic drives is that the Casimir force drive always yields an inward or attractive force, whereas the voltage on the electrostatic comb drive can be reversed in polarity, reversing the direction of the force. Another difference is that the Casimir force comb drive requires no



a) overlapping LxL conductive plates. b)comb structure with interleaved. L x L plates

c) conductive moveable piston (solid black line) in a conductive rectangular box L x L x 0.8L.

FIGURE 1 . Some structures with Casimir forces.

external power source, whereas the electrostatic drive does. One could imagine a second set of parallel plates with a variable spacing *a* to generate a force in the opposite direction. Instead of applying a variable voltage as in the electrostatic case, one would need to alter the spacing between the plates to control the direction and magnitude of the net force. This device could be operated as a motor if at least one set of plates had an adjustable spacing. Because of the conservative nature of the electromagnetic field, one would not expect to extract a net energy from the vacuum.

Casimir force comb type structures with parallel plates have many of the same design issues as electrostatic type comb structures. For example, the structures are unstable with respect to movement of the plates in the direction normal to the plate movement. In other words, the normal forces increase if one of the normal separation distances decreases.

Structures using rectangular cavities and sliding plates

The mechanical behavior of the parallel plate configurations is determined by the negative energy density that arises if $a \ll L$. If we consider cavities that have dimensions in orthogonal directions that are within about a factor of about 3 of each other, then we can have regions with positive or negative energy density and can obtain both attractive and repulsive forces on sliding plates.

For example, consider a rectangular cavity $L \times L \times a$ formed from conductive plates. Imagine that the side of length *a* is constructed so that we can insert an additional plate (assumed to have zero thickness) in a direction normal to the *a* direction, dividing the cavity into identical rectangular regions with sides $L \times L \times a/2$. By the conservation of energy we can compute the average force required to insert this moveable plate. If we assume that the vacuum energy density is altered only in the region within the cavity as we insert the plate, then the change in vacuum energy is equal to minus the average force $\langle F \rangle$ present during insertion of the movable plate times the distance L. Defining en(a1, a2, a3) as the vacuum energy of a rectangular cavity with sides (a1,a2,a3), we can express the average force as

$$\langle F(L,L,a)\rangle = - \, [2en(L,L,a/2) - en(L,L,a)]/L \; . \qquad (5)$$

Depending on the ratio of L/a we can obtain positive, zero, or negative average forces.

As discussed by (Maclay, 1999), the energy for a rectangular cavity is a homogeneous function of the dimensions: $en[l a1,\ l a2,\ l a3] = l^{-1}\ en[a1,a2,a3]$. With this information we can evaluate the average force for several examples. Assume a/L=0.816, then by numerical computation we have the final state en(L,L,0.408 L)=0, and for the initial state en(L,L,0.818L) = $0.1\nabla c/L$. For this geometry, the mean force is therefore:

$$\langle F(L,L,0.816L)\rangle = - \, [0 - 0.1\nabla c/L\,]/L = 0.1\nabla c/L^2 \; . \qquad (6)$$

This is a attractive force, pulling the moveable plate inwards with an average force approximately equal in magnitude to the force on a cube of side L. If we start with a cavity of zero energy, namely en(L, L, 3.4L) =0, then we have a final state with two cavities each with positive energy *en(L,L,1.7L) = 0.072Ñc/L* (Maclay, 1999). This configuration yields a positive or outward mean force during insertion of the plate equal to *<F(L,L,3.4L)>* = -*0.144Ñc/L*^{2} .

Rectangular structures with a moveable piston

Consider a rectangular conductive cavity *(L x L x a)* with a moveable piston that moves along the *a*-direction, dividing the cavity into two regions, each with its contribution to the total vacuum energy. We assume the piston is infinitely thin and normal to the *a*-direction (Fig. 1c). We can then numerically compute the total vacuum energy *Ep (L,L,x)* of the structure as a function of the distance x of the piston from one end of the cavity. From our definition of *en(a1,a2,a3),* and the definition *x = x/L* , we have

$$Ep(L,L,x) = [en(1,1, a/L - x) + en(1,1,x)]/L \quad . \tag{7}$$

If we differentiate this with respect to x, we obtain an expression for the force *F(x)* due to the vacuum stresses on the moveable plate. Consider an example in which *a = 0.8L,* so *0£ x £0.8.* Figure 2 shows the dimensionless energy and force respectively *LEp(L,L,x)* and $L^2F(x)$ as functions of *x*. For values of x near the center (x ≈ 8.0), the force on the piston is approximately directly proportional to x, and the energy is approximately a negative parabola with negative curvature. A small deflection from x=8.0 leads to a force causing an increased deflection. Thus Figure 2 shows the state of the system with the piston near the center is unstable: the piston would be pushed to the closest end of the cavity.

However, if we include the restoring force that arises from the small deflection of a deformable membrane as given by Hooke's Law, then this configuration might become stable if the material force constant exceeds that for the Casimir force. These results suggest the intriguing possibility of making a structure that displays simple harmonic motion for small displacements by employing two adjacent cubical cavities, with a common face that can be deflected.

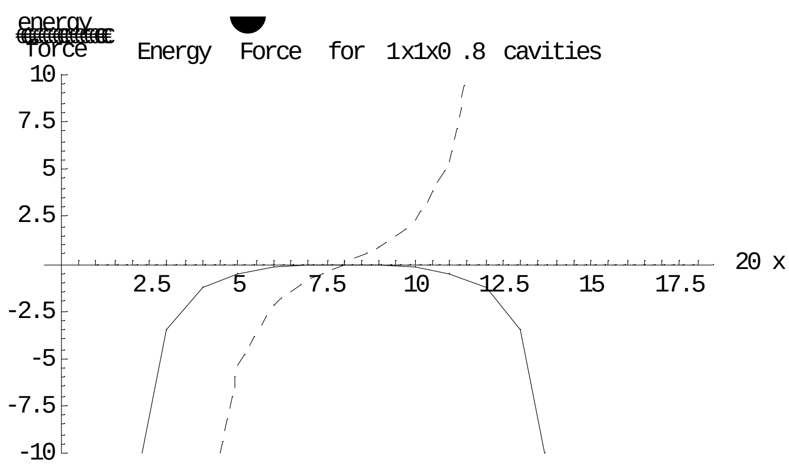


FIGURE 2. Plot of the vacuum energy (___solid line) and Casimir force (_ _ _ dashed line) for a 1 x 1 x .8 cavity that is divided into two rectangular cavities by a sliding piston that moves along the 0.8 direction. The maximum energy is at abscissa of 8 which corresponds to the center of the cavity, x = 0.4. For these calculations, we have set ∇*c*=1 and set L equal to 1 unit. To obtain a numerical result, we use the MKS value of ∇*c*. If we let L=0.5 micron, then the abscissa is in units of 0.025 micron, and the energy scale is in units of 6.3×10^{-20} joule and the force scale is in units of 1.26×10^{-13} joule. Forces of this magnitude are just measurable using AFM technology.

CONCLUSIONS

Some basic features of structures using parallel plates and rectangular cavities experiencing Casimir forces have been explored. Material properties need to be included in more detailed considerations, as well as corrections for finite temperatures. As the level of sophistication in MEMS fabrication increases, we may expect to see some of these structures realized. The utility of such devices is still unknown.

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