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The Role of Quantum Vacuum Forces in Microelectromechanical Systems

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Abstract

The presence of boundary surfaces in the vacuum alters the ground state of the quantized electromagnetic field and can lead to the appearance of vacuum stresses. In the last 5 years, landmark measurements of the vacuum stress between conducting uncharged parallel plates (Casimir force) have been made employing Atomic Force Microscopes. The AFM provides a highly accurate optical measurement of the deflection of a metallized sphere attached to the end of a micromachined cantilever under vacuum forces as small as about 10 picoNewtons. The sphere deflects due to the Casimir force as it is brought within about 20-700 nm of a flat surface. Recently the first micromachined MEMS (microelectromechanical system) device was fabricated that utilizes the Casimir force between parallel plates. The $1/d^4$ force dependence allows the device to serve as a highly sensitive position sensor. The are many other examples of quantum vacuum forces and effects besides the well known parallel plate Casimir force. Here we discuss potential roles of quantum vacuum forces and effects in MEMS systems and other systems. With the growing capability in nanofabrication, some of the roles may be actualized in the future. Because of the computational complexity, no theoretical results are yet available for a number of potentially interesting geometries and we can only speculate.

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I. INTRODUCTION

Zero-point field energy density is a simple and inexorable property of a quantum field, such as the electromagnetic field, which is a representation of the Lorentz group of transformations of special relativity. For a quantum field, the canonical position and momentum variables do not commute and consequently the lowest state of the field has a non-zero energy. For the electromagnetic field, if we assume the shortest wavelength photon to be included in the ground state spectrum has the Planck length of 10^{-35} m, then the predicted quantum vacuum energy density is quite large, about 10^{114} J/m^3 or, in terms of mass, 10^{95} g/cm^3. Such a large energy density is clearly a puzzling embarrassment to physicists, who for years routinely discarded this nearly infinite result in renormalization procedures.

However, there are measurable consequences of the zero point energy which arise because the ground state vacuum electromagnetic field has to meet the usual boundary conditions for the electromagnetic field. It is the effect of boundaries on the vacuum field that leads to the appearance of vacuum stresses, so called Casimir forces. Effects of this type occur for all quantum fields and can arise from the presence of surfaces as well as choices of topology of the space. Several approaches to computing electromagnetic Casimir forces have been developed that are not based on the zero point vacuum fluctuations directly. In the special case of the vacuum electromagnetic field with dielectric or conductive boundaries, various approaches suggest that Casimir forces can be regarded as macroscopic manifestations of many-body retarded van der Waals forces, at least in simple geometries with isolated atoms[1], [2]. Casimir effects have also been derived and interpreted in terms of *source* fields in both conventional [1] and unconventional [3] quantum electrodynamics, in which the fluctuations appear within materials instead of outside of the materials. Lifshitz provided a computation of the Casimir force between planar surfaces by assuming that stochastic fluctuations occur in the tails of the wavefunctions of atoms that leak into the regions outside the surface, and can lead to induced dipole moments in atoms in a nearby surface, which leads to an a net retarded dipole-induced dipole force between the planar surfaces[4]. These approaches differ in how they visualize the fluctuations of the electromagnetic field, but give consistent results in the few cases of simple geometries which have been computed[5]. It may be that these approaches display differences for computation of geometries with curvature, or for computations of the forces between separated curved surfaces[6].

The term "Casimir force" most commonly refers to the attractive vacuum force that exists between two parallel, infinite, conducting planes[7][8]. This attractive force, which is normal to the surface, arises because the surfaces change the mode distribution of the ground state quantized electromagnetic field. In the region between two parallel perfectly conducting plates, no modes with wavelengths larger than twice the separation can exist. We can also view this force as arising from radiation pressure, the transfer of momentum from the vacuum to the surfaces[9]. The Casimir effect was first predicted in 1948, but was not measured accurately until the last few years[10][11]. Corrections for finite conductivity and surface roughness have been developed for the parallel plate geometry, and the agreement between theory and experiment is now at the 1% level or better for separations of about 0.1-0.7 μm [12]. In actual practice, the measurements are frequently made with one surface curved and the other surface flat, and the proximity force theorem, which is under scrutiny currently, is used to account for the curvature. This experimental approach eliminates the difficulties of trying to maintain parallelism at submicron separations. Mohideen and collaborators have made the most accurate measurements to date in this manner, using an AFM (Atomic Force Microscope) that has a metallized sphere about 250 μm in diameter attached to the end of a cantilever about 200 μm long, capable of measuring picoNewton forces. The deflection of the sphere is measured as it is moved close to a flat metallized surface[10]. The more difficult measurement between two parallel plates has been made and shown to give results that are consistent with theory[13]. Measurements of the force between two parallel surfaces each with a small (1 nm) sinusoidal modulation in surface height, have showed that there is a lateral force as well as the usual normal force when the modulations of the opposing surfaces are not in phase [14].

Parallel plate Casimir forces go inversely as the fourth power of the separation between the plates. The Casimir force per unit area F between perfectly conducting plates equals $F = -\pi^2 \hbar c/240 d^4$ and is equivalent to about 1 atm pressure at a separation of 10 nm, and so is a candidate for actuation of MEMS (MicroElectroMechanical Systems). In MEMS, surfaces may come into close contact with each other, particularly during processes of etching of sacrificial layers in the fabrication process. In 1995 the first analysis of a dynamic MEMS structure that used vacuum forces was presented by Serry et al[15]. They consider an idealized MEMS component resembling the original Casimir example of two parallel plates, except that one of the plates is connected to a stationary surface by a linear restoring force and can move along the direction normal to the plate surfaces. The Casimir force between the two plates, together with the restoring force acting on the moveable plate, results in an "anharmonic Casimir oscillator" (ACO) exhibiting bistable behavior as a function of the plate separation. The analysis also demonstrates that the Casimir effect could be used to actuate a switch, and might be responsible in part for the "stiction" phenomenon in which micromachined membranes are found to latch onto nearby surfaces.

Smaller distances between MEMS components are desirable in electrostatic actuation schemes because they permit smaller voltages to be used to generate larger forces and torques. MEMS currently employed in sensor and actuator technology generally have component separations on the order of microns, where Casimir effects are negligible. Casimir effects will be of increasing significant in microelectromechanical systems (MEMS) as further miniaturization is realized [15]. An experimental realization of the ACO in a nanometer-scale MEMS system has recently been reported [16]. In the experiment the Casimir attraction between a 500 μm-square plate suspended by torsional rods and a gold-coated sphere of radius 100 μm was observed as a sharp increase in the tilt angle of the plate as the sphere-plate separation is reduced from 300 nm to 75.7 nm. This "quantum mechanical actuation" of the plate suggests "new possibilities for novel actuation schemes in MEMS based on the Casimir force" [16]. In a refinement of this experiment, a novel proximity sensor was demonstrated in which the plate was slightly oscillated with an AC signal, and the deflection amplitude observed gave an indication of the precise location of the nearby sphere[17]. A measurement using a similar torsion oscillator was recently reported using gold on the sphere and chromium on the plate[18].

II. LIMITATIONS OF CURRENT THEORETICAL CALCULATIONS OF VACUUM FORCES

The parallel plate geometry (and the approximately equivalent sphere-plate geometry or sphere-plate with small deviations geometry) is essentially the only geometry for which experimental measurements have been conducted and the only geometry for which the vacuum forces between **two separate surfaces** (assumed to be infinite) have been computed. Vacuum forces are know to exist in other experimental configurations between separate surfaces, but rigorous calculations based on QED (quantum electrodynamics) are very difficult and have yet to be completed[6]. Since it is experimentally possible to measure forces between various separate surfaces, with the improvement in experimental techniques, theoreticians may soon see the need for such computations.

Calculations of vacuum stresses for a variety of geometric shapes, such as spheres, cylinders, rectangular parallelepipeds, and wedges are reviewed in [7][8]. In general, calculations of vacuum forces become very complex when the surfaces are curved, particularly with right angles. Divergences in energy appear, and there are disagreements about the proper way to deal with these divergences[19]. In addition, in the usual calculations, only a spatial average of the force for a given area for the ground state of the quantum vacuum field is computed, and all material properties, such as binding energies, are ignored, a procedure which Barton has questioned recently[20][21][22]. Computation has shown that the vacuum stress on a spherical metal shell, a cubical shell, or a solid dielectric ball is a uniform force is repulsive, or directed outward.

Because of the very special nature of the parallel plate geometry and the high degree of symmetry of the cube and sphere, it is not reliable to make generalizations about the behavior of vacuum forces based on these special geometries. The vacuum forces on the faces of conductive rectangular boxes or cavities show very different features compared to those of the parallel plate, the cube, and the sphere. For a rectangular parallelepiped cavity, the total force on a given face (the differential force integrated over the entire face) can be positive, zero, or negative depending on the ratio of the sides of the box[22][23][24][25]. In fact there are cavities that have zero force on two sides and a positive or negative force on the remaining side. The are boxes for which the energy is negative (or positive) and the forces on some walls are attractive while the forces on the remaining walls are repulsive. Indeed it is difficult to get an intuitive picture of the meaning of these results.

From a technological viewpoint, it would be useful to be able to generate repulsive vacuum forces as well as attractive vacuum forces. From a fundamental viewpoint, it is unclear how one can have a repulsive force in vacuum if the force can be correctly modeled as a dipole-induced dipole force. Thus there is great interest in measuring the vacuum forces in different geometries that are predicted to be repulsive. However, there is no easy way to measure vacuum forces on spheres or rectangular cavities[26]. One might consider applying a stress to the spherical shell, and observe the deformation. This is a difficult experiment since the sphere would probably have to be submicron in diameter for the Casimir force to be large enough to be measurable. Further, the deflection measured would be measured would depend on the properties of the material of which the sphere was made, and such properties are not included in the usual calculations of the Casimir force[21]. Alternatively one might contemplate cutting a sphere in half, and measuring the force between the two hemispheres using an Atomic Force Microscope. However, the question arises: If we cut a spherical cavity into two hemispheres, will we find a repulsive force between the two separate surfaces? Or will an attractive force between the edges dominate? No computations have yet been done for this situation for real materials. For optically thin materials Barton shows the net force will be attractive[20][21].

Vacuum forces computed for a perfectly conducting cube with thin walls are also repulsive or outward, and experimentalists have the same conundrum regarding the meaning of this calculated vacuum force. To measure the force one might imagine freeing one face of the cube, and then moving it very slightly normal to its surface, in the spirit of the principle of virtual work dE=-Fdx. Unfortunately no one has computed the force between a cube with one side removed and a nearby surface which is parallel to the missing face. We have attempted to measure the force between an array of open cavities (wall thickness about 150 nm, cavity width about 200 nm) and a metallized sphere 250 μm in diameter on an AFM cantilever, and to date have only observed attractive forces[27].

Another limitation of the calculations to date for the rectangular cavity, is that only the total force on each face is computed. The differential vacuum stress in not uniform on each wall, and, in order to avoid issues with divergences, the differential force is integrated over the face. How these nonuniformities might affect experiments is unknown.

III. VACUUM ACTUATED MEMS SYSTEMS

We consider a variety of systems whose function is based on present calculations of the properties of the ground state of the quantum vacuum. Several different potentially interesting applications are considered in [5] No experimental investigations have yet been conducted on most of these systems that we are aware of.

A. Vacuum Forces on Particles

The parallel plate vacuum forces have been extensively measured and calculated, and even utilized in a sensitive position sensor. The question arises: what other manifestations of vacuum forces will be of technological interest as the dimensions of MEMS devices are reduced? We mention a few examples that may be of interest. The first has to do with forces on charged or polarizable particles in the vacuum. The electromagnetic field of the ground state of the quantum vacuum shares the properties of the fields for excited states of the electromagnetic field, when real photons are present. Whenever there is an inhomogeneous vacuum energy density, there will a net force on a polarizable neutral particle given by $\frac{1}{2}\alpha\overrightarrow{\nabla}\langle E(x)^2\rangle$. Local changes in mode density and therefore vacuum energy density are induced by the presence of curved surfaces, and, depending on whether the curvature is positive or negative, the force between the surface and the particle may be repulsive or attractive [19]. The simplest example of a surface altering the vacuum modes is a perfectly conducting, infinite wall. The change in the vacuum field energy due to the wall produces in this case the well-known Casimir-Polder interaction: for sufficiently large distances d from the wall this interaction is $V(d) = -3\alpha\hbar c/8\pi d^4$, where $\alpha(0)$ is the static polarizability of the (ground-state) atom. This effect has been accurately verified in the elegant experiments of Sukenik *et al* in which he measured the deflection of an atomic beam near a surface[28]. In this experiment, the particles are actually passed between the surfaces of a wedge, two conducting planes that intersect at an angle β radians. The stress-energy tensor is not constant in this region, as it is between two parallel plates, but T^{00} increases as one moves closer to the point of intersection, at which there is a singularity. In the experiment, only the effect of the force approximately normal to the surfaces was measured. As one might expect, there is also a radial force on a particle at a distance r and at an angle $\beta/2$ from the intersection that tends to accelerate the particle toward the intersection provided the static polarizability α is positive[29]:

$$F_r(r) = -\frac{\alpha(0)\hbar c}{90\pi r^5\beta^4}(44\pi^4 + 80\pi^2\beta^2 + 11\beta^4)$$

For the case of $\beta = \pi$ we have a particle near a plane and recover the usual Casimir-Polder force. The tangential force in the θ direction vanishes along this midline. Note that there would be a torque on a permanent dipole in this wedge.

There are many geometries for which the stress-energy tensor has not been computed as a function of position, and we do not know what the forces on a charged or polarizable particle in the vacuum might be. Consider for example, the forces on a particle within a closed rectangular cavity, where the kinetic energy of the particle is much less than the change in vacuum energy due to the surfaces. Very near any surface, away from edges and other walls, one might expect the particle to experience the usual Casimir-Polder force. In other regions of the cavity the forces are not know since calculations of the stress-energy tensor have not been done without averaging over the entire volume. What is the equilibrium state of a group of atoms or particles in a region of altered vacuum energy? For example, assume we have a number of particles in a metal sphere or a metal box in which the vacuum modes have been altered from the free field modes. What is the equilibrium distribution of these particles? Since there is a non-homogeneous vacuum field, the particles will experience forces. Will the particles keep accelerating due to these forces or will there be some vacuum damping that gives them a terminal velocity? Will the particles congregate in a region of the lowest energy? Will they bounce off the walls and give some kind of force on the walls. Are these forces negligible, except at very low temperatures? The motion of one particle inside a box or sphere would be interesting. Does the interaction provide a window into vacuum energy so that we can make two reservoirs to operate an engine?? If a hole is put in one of the sides of the box, what happens? There is one calculation that suggests that very high energy particles observed in space may derive their kinetic energy from a long term acceleration due to the stochastic vacuum field[30].

B. Systems with Torques

Consider the conditions for which we would expect a medium, such as a dielectric slab, to experience a torque in the vacuum. If we view the origin of a vacuum torque as the transfer of the angular momentum of zero-point photons to the medium, then it is clear to have a torque we need to have a geometrical configuration in which the vacuum energy depends on the angular orientation of the medium. This requirement cannot be met with a single object, even if it is not isotropic. However, two plates separated by a distance d that are birefringent would break the rotational symmetry of the vacuum and be expected to experience a torque. This torque has indeed been calculated, and compared to the attractive

Casimir force between the plates[31]. From dimensional grounds the torque between two thick plates (thickness>>separation) of area A goes as $f\hbar cA/d^3$, where Enk has derived an expression for the dimensionless number f which is determined by the square of the difference of the refractive indices, and has a typical value of about 10^{-6}. The torque, which appears measurable, varies as $\sin 2\phi$, where ϕ is the angle between the two optic axes. The dielectrics tend to rotate in opposite directions so the total momentum transfer form the vacuum is zero.

C. Forces on Semiconductor Surfaces

One of the potentially most important configurations from the technological viewpoint involves vacuum forces on semiconductor surfaces. The Casimir force for a conducting material depends on the plasma frequency, beyond which the material tends to act like a transparent medium. For parallel plates separated by a distance d the usual Casimir force is reduced by a factor of approximately $C(a) = (1 + (8\lambda_p/3\pi d))^{-1}$, where λ_p is the wavelength corresponding to the plasma frequency of the material[32]. Since the plasma frequency is proportional to the carrier density, it is possible to tune the plasma frequency in a semiconductor, for example, by illumination or by temperature, or by the application of a voltage bias. In principle it should be possible to build a Casimir switch that is activated by light, a device that would be useful in optical switching systems. A very interesting measurement of the Casimir force between a flat surface of borosilicate glass and a surface covered with a film of amorphous silicon was done in 1979 by Arnold et al[33] They observed an increase in the Casimir force when the semiconductor was exposed to light. This experiment has yet to be repeated with modern methods and materials.

D. Vibrating Cavity Walls in MEMS Cavities

The unexpected behavior of forces on the walls of a rectangular cavity mentioned previously allows us to model a cavity with dimensions such that a wall vibrates in part due to the vacuum stress. For example, a cavity that is 2 μm long, 0.1 μm wide, and about 0.146 μm deep will have zero force on the face normal to the 0.146 direction. The zero force corresponds to an unstable energy maximum. Thus a deflection inward leads to an increasing

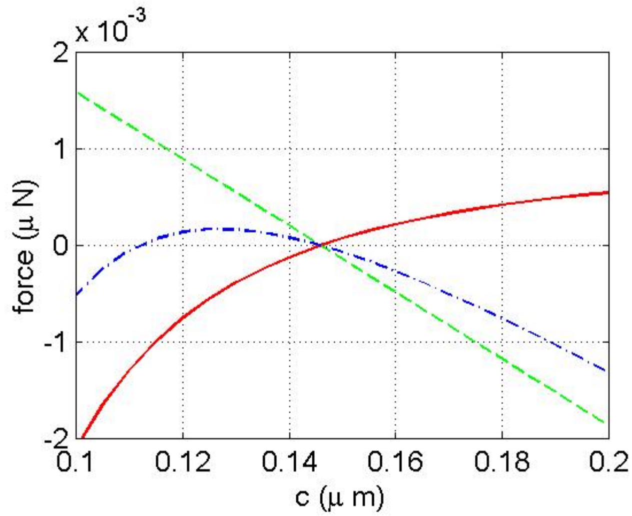

FIG. 1: The force on the top surface of a closed, perfectly conducting rectangular cavity 2 μm long by 0.1μm wide, as a function of the depth c. The equilibrium position is c_{eq}=0.146 μm. The dashed line (- - -) is a plot of the linear restoring force from a silicon spring as a function of the deformation of the top of the box, assumed to be made of silicon; the solid line (—) is the destabilizing vacuum force on the top of the box; and the dot-dash line ($- \cdot -$) is the total force on the top of the box. Note: The force on the y-axis is actually the total force for 1000 boxes.

inward (attractive) force, and, conversely, any deflection outward (repulsive force) leads to an increasing outward force. This potential is akin to a harmonic oscillator, except the force is destabilizing (F=kx) rather than stabilizing (F=-kx). If we assume that the box is made of real conductive materials, then there will be a restoring force due to the material. If we include the restoring force that arises from the small deflection of a deformable membrane as given by Hooke's Law, then this configuration might become stable if the material force constant exceeds that for the Casimir force (Fig. 1).

These results suggest the intriguing possibility of making a structure that displays simple harmonic motion for small displacements with a frequency that depends on the difference of the material force constant and the vacuum force constant. The face of a box of the proper dimensions may oscillate under the mutual influence of the vacuum force and the Young's modulus of the material (Fig. 2a). The oscillations would be damped due to the non-ideal properties of the material and the friction with the environment (Fig. 2b). A zero point oscillation of the cavity wall would be expected. The energy in the lowest mode would be modified by the temperature.

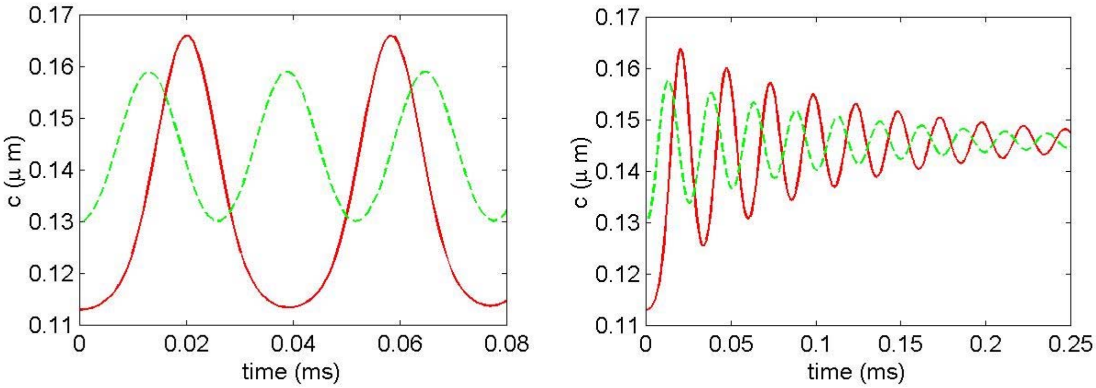

Figure 2. a) Displacement of the cover plate as a function of time for two starting positions. The solid curve is for an initial deflection from the equilibrium position to a spacing of 0.113 μm, close to the minimum for oscillatory behavior, and clearly shows anharmonic behavior. The dashed is for a smaller initial offset from the equilibrium position, and results in a more sinusoidal motion. b) Displacement vs. time for the same two initial displacements, but including a damping ratio of 0.025.

E. Extraction of Energy from the Quantum Vacuum

The question naturally arises: If QED predicts a large energy density in the quantum vacuum, is there some way to make us of this vast energy? From the scientific viewpoint, the answer seems clear that it is possible to extract energy from the vacuum. However the process as currently understood does not appear to have much practical application. To illustrate, consider, an arrangement of two perfectly conducting, uncharged, parallel plates in a vacuum as an energy source. The Casimir energy $U_C(x)$ at zero degrees Kelvin between plates of area A, separated by a distance x is:

$$U_C(x) = -\frac{\pi^2}{720}\frac{\hbar c A}{x^3} \tag{1}$$

If we allow the plates to move from a large initial separation a to a very small final separation b then the change in the vacuum energy energy between the plates is approximately:

$$\Delta U_C = U_C(b) - U_C(a) \tag{2}$$

$$\approx -\frac{\pi^2}{720}\frac{\hbar c A}{b^3} \tag{3}$$

The attractive Casimir force has done work on the plates, and, in principal, we can build a device to reversibly extract this energy and use it. At the end of the motion ($x = b$), the energy of the electromagnetic field of the quantum vacuum between the plates has been reduced by the amount of the work done, so, as is necessary, the total energy is conserved.. In practice the closest distance in separation is about a nanometer due to surface roughness. However, in practice, the forces are piconewtons over a distance of nanometers, so very little useful energy is extracted. In addition, once the plates have moved together, and the energy has been extracted, one has to do the same amount of work to separate the plates and return them to the initial positions since this is a conservative system[34]. In the future more practical methods of extracting energy from the quantum vacuum may be developed.

It is important to reiterate that utilizing energy of the quantum fluctuations of the electromagnetic field does not appear to directly violate known laws of physics, based on the work of Forward and Cole and Puthoff, however improbable or impossible such a development might seem [35][36]. Forward showed that it is possible to conceive of a device, a foliated capacitor, in which one could extract electrical energy from the quantum vacuum to do work. The energy is extracted as the portions of the capacitor that repel each other due to electrostatic forces come together under the influence of the Casimir force[35]. Cole and Puthoff used stochastic electrodynamics to examine the process of removing energy from the vacuum fluctuations at zero temperature from the viewpoint of thermodynamics and showed there is no violation of the laws of thermodynamics[36]. In the same spirit, Rueda has suggested that very high energy particles observed in space may derive their kinetic energy from a long term acceleration due to the stochastic vacuum field[30]. In a careful analysis, Cole has shown that this process of energy transfer from the vacuum field to kinetic energy of the particles does not violate the laws of thermodynamics[37]. In stochastic electrodynamics one treats the vacuum fluctuations as a universal random classical electromagnetic field. A formal analogy exists between stochastic electrodynamics and quantum electrodynamics: the field correlation functions in one theory are related to the Wightman functions in the other theory[38]. Pinto has done a calculation of a solid state Casimir device which is described as a transducer of vacuum energy which can operate in a repetitive cycle [39].

F. Vacuum Powered Space Craft

It is possible to conceive of a vacuum spacecraft that operates by pushing on the quantum vacuum[40]. With a suitable trajectory, the motion of a mirror in vacuum can excite the quantized vacuum electromagnetic field with the creation of real photons. This possibility was first noticed in 1970, when Moore considered the effect of an uncharged one dimensional boundary surface in vacuum that moved, with the very interesting prediction that it should be possible to generate real photons from a suitable motion[41][42]. Energy conservation requires the existence of a radiation reaction force working against the motion of the mirror[43]. The energy expended moving the mirror against the radiative force goes into electromagnetic radiation. This effect, generally referred to as the dynamic or adiabatic Casimir effect, has been reviewed [7][8][44]. The vacuum field exerts a force on the moving mirror that tends to damp the motion. This dissipative force may be understood as the mechanical effect of the emission of radiation induced by the motion of the mirror.

The vacuum radiation pressure and the radiated spectrum for a non-relativistic, perfectly reflecting, infinite, plane mirror was computed by Neto and Machado for the electromagnetic field in three dimensions, and shown to obey the fluctuation-dissipation theorem from linear response theory[43][45]. This theorem shows the fluctuations for a stationary body yield information about the mean force experienced by the body in nonuniform motion.

Assume we have a flat, perfectly reflecting, mirror whose equilibrium position is $x = 0$. At a time t where $t_i < t < t_f$ the location of the mirror is given by $x(t)$. Neto has given an expression for the force per unit area $F(t)$ on such a mirror[45]:

$$F(t) = \lim_{\delta x \to 0} \frac{\hbar c}{30\pi^2} \left[\frac{1}{\delta x} \frac{d^4 x(t)}{c^4 dt^4} - \frac{d^5 x(t)}{c^5 dt^5} \right] \tag{4}$$

where δx represents the distance above the mirror at which the stress-energy tensor is evaluated. The second term represents the dissipative force that is related to the creation of travelling wave photons, in agreement with its interpretation as a radiative reaction. In computing the force due to the radiation from the mirror's motion, the effect of the radiative reaction on $x(t)$ is neglected in the nonrelativistic approximation. The divergent first term can be understood in several ways. Physically it is a dispersive force that arises from the scattering of low frequency evanescent waves. The divergence can be related to the unphysical nature of the perfect conductor boundary conditions. Forcing the field to vanish on the surface requires its conjugate momentum to be unbounded. Thus the average of the stress-energy tensor $\langle T_{\mu\nu}\rangle$ is singular at the surface for the same reason that single-particle quantum mechanics would require a position eigenstate to have infinite energy[46]. This divergent term can be lumped into a mass renormalization, and therefore disappears from the dynamical equations when they are expressed in terms of the observed mass of the body[47][48]. We will not discuss this term further. We will assume that diffractions effects are small for our finite plates.

Assume that the mirror is vibrated harmonically so its displacement is $X_o \sin \Omega t$. The total impulse $I_m = \int F dt$ per area per cycle for our model trajectory would vanish since the force is positive during the first and fourth quarters of a cycle, and negative during the second and third quarters. If we imagine altering the cycle during the second and third quarters, so that the motion is given by a cubic polynomial of time in the middle half of the cycle, then there would be no contribution to the force during this period and the net acceleration is

$$a = -\frac{\hbar}{15\pi^2} X_o (\frac{\Omega}{c})^4 \frac{\Omega}{M} \tag{5}$$

where M is the mass per unit plate area of the spacecraft, and we assume the plate is the only significant mass in the gedanken spacecraft. In order to estimate a, we can make some very favorable assumptions regarding the mass per unit area of the plates $M = m_p/a_o^2$, (m_p is proton mass, a_o is Bohr radius), and assume some reasonable numerical values (frequency $\Omega = 3x10^{10}\,\mathrm{s}^{-1}$; oscillation amplitude $X_o = 10^{-9}\,\mathrm{m}$) we find that a is approximately $3x10^{-20}\,\mathrm{m/s}^2$ per unit area, not a very impressive acceleration[49][50]. Making the plate part of a cavity increases the photon emission by a factor of 10^9, improving the performance slightly[49][50]. Use of new materials with increased strength, such as perfect crystals or new alloys with dislocation-free plastic deformation that exhibit "super" properties could provide further improvements. An amplitude of oscillation of 1 mm would yield velocities comparable to those achieved by a chemical rocket.

The point of this computation is not to suggest a practical way to build a spacecraft, but to illustrate a potential role of the quantum vacuum. Perhaps a more clever quantum drive will some day become practical or other uses of the dynamic Casimir effect will arise. Physicists have explored various means of locomotion depending on the density of the medium and the size of the moving object. It would be interesting to find an optimum method for moving in the quantum vacuum. Unfortunately we currently have no simple way to mathematically explore various simple possibilities.

IV. CONCLUSION

The are many potential ways in which the ground state of the vacuum electromagnetic field might be exploited in technological applications, a few of which we have mentioned here. As the technology to fabricate small devices improves, as the theoretical capability of calculating quantum vacuum effects improves, it will be interesting to see which possibilities prove to be useful and which just remain curiosities. It a way the situation is reminiscent of electricity in the 1600s, when Faraday was asked of what use is electricity?, and answered "Of what use is a new born baby?". We are not very good at predicting the development of technology. In the 1960s, manufacturers were hard put to think of any reason why an individual would want a home computer and today we wonder how we ever survived without them.

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