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Unusual Properties of Conductive Rectangular Cavities in the Zero Point Electromagnetic Field: Resolving Forward's Casimir Energy Extraction Cycle Paradox

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Abstract. Two infinite, parallel, uncharged conducting planes experience an attractive force between them (called the Casimir force) due to the alteration of the zero point electromagnetic field between the plates. Similarly, there are forces on the surfaces of a rectangular cavity with conductive walls of dimension a_1, a_2, a_3. Recently a paradox was published describing a method for the extraction of mechanical energy from the zero point fluctuations of the electromagnetic field in a rectangular conductive cavity by cyclical changes in the dimensions of the walls without doing any work (Forward, 1998). The validity of the analysis depends on the implicit assumption that the energy density within the cavity is approximately isotropic, so that positive average energy densities within the cavity result in outward forces, and negative average energy densities result in attractive forces. However, detailed computations of the forces on the cavity walls show this assumption is not valid, and that there are positive energy regions in which there are outward forces on some faces and inward forces on other faces (Hacyan, 1993). Specifically, for a cavity with $a_1 = a_2 = 1$ the energy is positive for $0.4<a_3<3.3$, however, the average pressure P_1 on the 1 x 1 faces and the average pressure P_3 on the 1 x a_3 faces are both positive <u>only</u> if $0.7<a_3<1.6$. For all other values of a_3, P_1 and P_3 have opposite signs. Specifically, for $a_3>1.6$, $P_3>0$, $P_1<0$ and for $a_3<0.7$, $P_3<0$, $P_1>0$. The implications of these and other unusual features of rectangular cavities in the vacuum are discussed.

INTRODUCTION

Quantum Electrodynamics predicts a non-zero expectation value for the energy of every mode of the electromagnetic field, even when no charges are present. The presence of conductive surfaces alters the boundary conditions that the source free fluctuating electromagnetic field must meet. The boundary conditions determine the modes of the vacuum field that are present, and hence the electromagnetic energy and momentum density. The vacuum forces result because the energy and momentum density are altered from the free field values.

At submicron separations, the fluctuations in the ground state of the quantized electromagnetic field lead to significant forces between components of a microsystem. For example, at a separation of 100 A, the predicted Casimir force between two perfectly conducting parallel metal plates in a vacuum is equivalent to 1 atmosphere pressure. A recent measurement gave forces to with 5% of Casimir's predictions (Lamoroux, 1997). Casimir forces have also been observed between dielectrics (Israelachvili, 1972). Using Casimir forces to extract energy from the vacuum is possible from both the experimental and theoretical viewpoints (Forward, 1984; Cole, 1993). It is expected that the total energy of the system (including the surface energy) is conserved in this process.

In order to understand the meaning of the force computations for cavities, we need to clarify the methods used.

The Casimir energy for a particular geometry, e.g. a cavity or parallel plates, is obtained as the difference between the vacuum energy E_o with no surfaces present (or the surfaces at infinity) and the vacuum energy E_s with the surfaces present (Plunien, 1986). Because modes with arbitrarily short wavelengths are present, both E_o and E_s are divergent quantities, however the difference $E_s - E_o = E_c$ is the finite Casimir energy.

To secure a finite Casimir energy E_c, either the contributions of the poles cancel (Brown, 1969) or various methods are used to deal with the infinite quantities, including wavelengths cut-offs, regularization of poles (Ambjorn, 1983), or the extraction of poles by use of Fourier transforms (Hacyan, 1993). Physically, the presence of surfaces changes the type and number of modes of the vacuum electromagnetic field present in the system. In general the

surfaces will tend to eliminate EM modes with wavelengths that are longer that the characteristic separations between opposing surfaces. For real conducting surfaces there is a further limitation: the surfaces will not be effective boundary conditions for wavelengths corresponding to frequencies which are above the plasma frequency of the metal. In any case, the higher frequency modes with frequencies above the plasma frequency, or with wavelengths much less than the characteristic separation between the surfaces, are not affected by the surfaces. It is these higher energy modes that give the divergent energies (or very large energies if a cut off, such as the Planck length or the Compton radius of the electron is used). Thus surfaces do not eliminate the high frequency divergences in energy seen in quantum electrodynamics. Casimir energies result from the changes in the lower frequency modes due to the presence of conducting surfaces altering the boundary conditions for the electromagnetic field.

Rigorous computations of the changes in the ground state vacuum energy density that are produced with different geometries have shed some light on the interpretation of the vacuum energy density and the Casimir force. Often the explanation for the attractive force between two perfectly conducting, parallel plates is that the energy density between the plates is less than the "free field" energy density outside the plates because wavelengths longer than twice the plate separation are excluded. Therefore, the plates attract each other because movement toward each other would reduce the free energy of the system.

Detailed QED calculations have shown that this intuitive model of the force due to vacuum fluctuations is probably misleading in its simplicity (Plunien, 1986). For example, excluding modes does not always lead to reduced energy density. In the case of a cube, the energy density increases as the side is reduced. For a rectangular cavity, there are configurations with negative average energy in which some walls have positive (outward) forces while other walls have negative (inward) forces. Rigorous field theoretic computations predict that both the magnitude and direction of the force due to the ground state of the electromagnetic field are strongly dependent on the specific geometry, and the force is repulsive in several cases.

Table 1 shows the variation in magnitude and sign of the energy density E_c for several simple geometries. The forces on the surfaces of these geometries are usually obtained by differentiating the energy densities according to the principle of virtual work:

$$dE_c = -F\,da \quad , \tag{1}$$

where F is the force in the "a" direction due to the zero point field. This computation of force tacitly assumes that the energy density is isotropic, so that the pressure is uniform over the surfaces in question. Because of the symmetric nature of these geometries this assumption appears valid for these cases, however, for rectangular cavities in which the sides are finite and not equal, the assumption of isotropic energy density is incorrect. Forces are best computed by using a more fundamental formulation, for example, by computing all components of the stress-energy tensor and integrating over surfaces (Brown, 1969; Hacyan, 1993). In the few cases for which independent computations are available, it appears that it is also possible to compute forces correctly by differentiating E_{ac}, the Casimir energy per unit area, where the area is on a surface normal to the displacement.
The force/area or pressure P_x in the x direction equals the partial derivative of the E_{ac}

$$P_x = -\partial E_{ac} / \partial x \quad . \tag{2}$$

This formula must be applied with caution. For example, for a rectangular cavity, when the energy E_c is zero, the derivative is not zero.

TABLE 1. Change in Vacuum Energy Density E_c for Different Perfectly Conductive Geometries

Parallel Plate (spacing =a)	Cube (side=a)	Sphere (radius=r)
$-0.0137hc/2\pi\, a^4$	$+0.092hc/2\pi\, a^4$	$+0.01\,07hc/2\pi\, r^4$

CASIMIR ENERGY DENSITIES AND FORCES IN RECTANGULAR CAVITIES

Although the results in Table 1 have been calculated by a variety of methods, we have very little real understanding about the role of geometry in determining the vacuum stresses. The lack of understanding is underscored by the unusual behavior of the energy density in a parallelopiped.

As the relative dimensions change from a cube into the usual parallel plate geometry, the average energy density changes from positive to negative. Figure 1 shows a contour plot of the electromagnetic average Casimir energy density Ec/V for a perfectly conducting box with sides a_1 x a_2 x a_3 computed by (Ambjorn, 1983). The figure shows lines of constant average energy as a function of the ratio of the sides a_2/a_1 and a_3/a_1. In the region with $a_1 = a_2 = a_3$ (near the origin) the average energy density is positive and a maximum for the cubic box. When one dimension, e.g. a_2, becomes equal to about $3.3a_1$ (a_2/a_1=3.3), then the average energy density E_c/V is zero, meaning it is the same as the free field energy density with no boundaries. The zero energy line is shown as a thick curve. The parallel plate Casimir force as computed by Casimir corresponds to the asymptotic case where both a_3/a_1 and a_2/a_1 are very large, and the energy density is negative.

The forces on different faces of a rectangular 1 x 1 x C cavity have been computed (Hacyan, 1993). Figure 2 shows the Casimir energy E, average energy density E/V, the average pressure P_3 /on the 1 x 1 face, and the average pressure P_1 on the 1 x C faces, all as functions of the dimension C (Hacyan, 1993). On this figure, consider the point corresponding to a cube with unit sides (C=1). The pressure $P_1 = P_3$ is the same on all faces by symmetry, and by calculation it is positive or outward.

Assume we let one side of the cube expand so one square face moves away from the opposite square face a distance C. Work will be done by the Casimir energy in the cavity since the pressure P_3 on this face is outward and almost a constant for this experiment. As C is increased, the pressure P_1 on the face with side 1 x C decreases and becomes negative or attractive at about C = 1.7. P_1 becomes more negative as C continues to increase. If we let this side continue to increase in length until it is about 3.3 times the other dimensions, the total energy in the cube has reduced to zero, the same result as given by (Ambjorn, 1983). Since this strange 1 x 1 x 3.3 cube has zero total

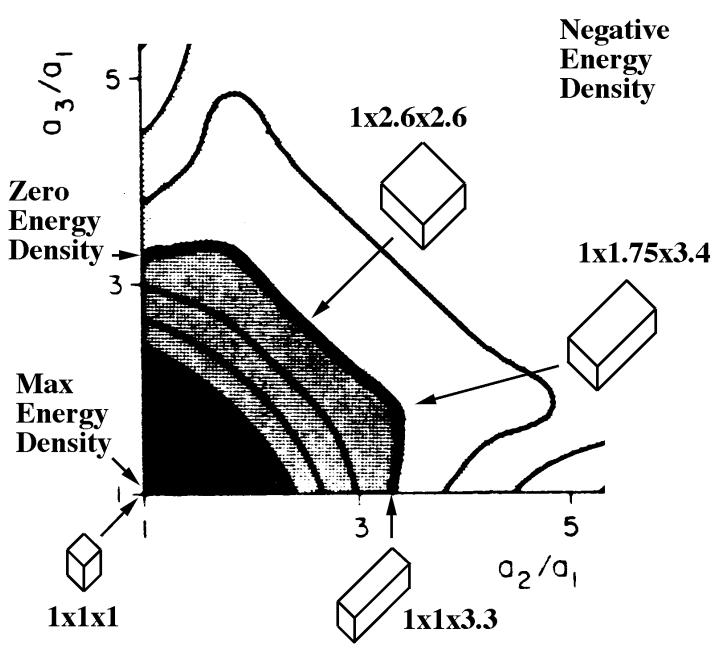

FIGURE 1. Constant energy curves are shown for a perfectly conducting rectangular cavity a x a x a. The thick line is a zero energy curve. (Forward, 1998)

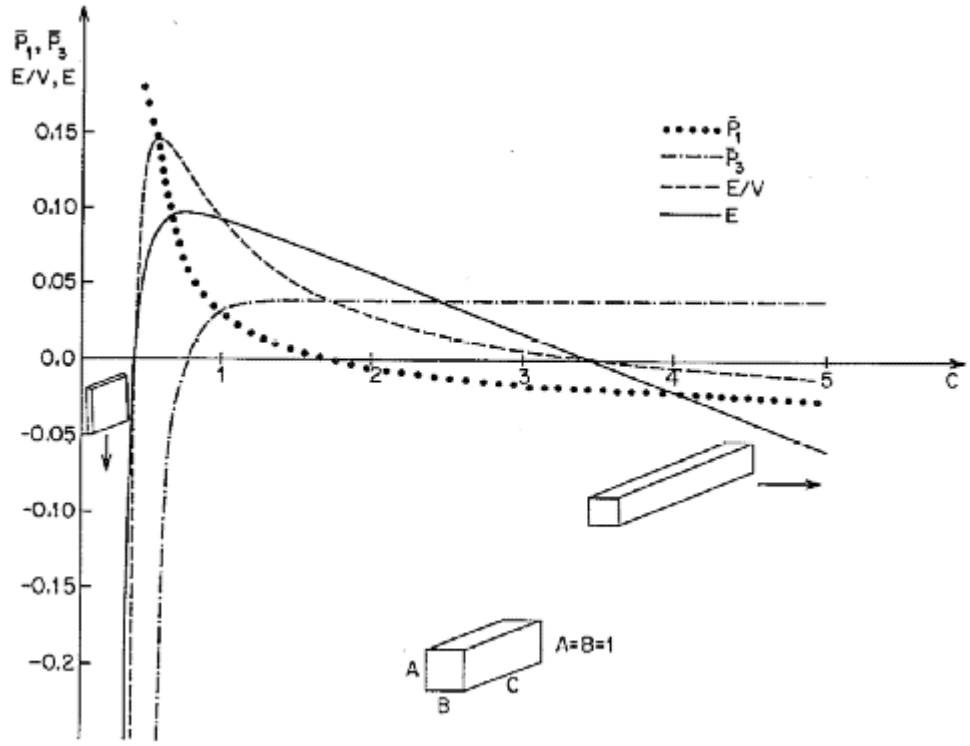

FIGURE 2. For a perfectly conducting rectangular 1 x 1 x C cavity, the graph shows the Casimir energy E, average energy E/V, the average pressure P_1 on the 1 x C faces, the average pressure P_3 on the 1 x 1 faces. (Hacyan, 1993).

energy, and zero average energy density, one might think that the forces on the walls would be zero. However, they are not zero because the **derivative** of the energy per unit area with respect to the displacement of the sides is not zero.

For this zero energy configuration of a bread box 1 x 1 x 3.3, Figure 2 shows that the pressure P_3 on the 1 x 1 side that moved still remains positive, but the pressure P_1 on all the 1 x 3.3 sides has become negative, or attractive. This suggests that the total energy is zero because we have sides with positive or outward pressures as well as sides with negative or inward pressures. Thus the work to assemble such a cavity would have positive contributions that cancel the negative contributions.

Indeed, imagine if we were to expand the other sides of the 1 x 1 x 3.3 box so that a cube of 3.3 on a side results. This process would have to increase the energy since we know it is positive for a cube, with positive forces on all faces. To reach this final configuration the forces on the 1 x C sides, which are initially negative, must change sign as we approach the geometry of a cube. Conversely, as Figure 2 shows, if we start with the unit cube 1 x 1 x 1 and decrease C, we find that P_3 decreases to zero at about C = 0.7, where dE/dC vanishes, and becomes negative, while P_1 increases rapidly.

In effect the energy density is not isotropic and the derivative of the energy density is not isotropic, as we usually assume it is. Thus we cannot conclude that the force on a cavity with a positive average energy density is necessarily outward or positive.

FORWARD'S PROPOSED CASIMIR ENERGY EXTRACTION CYCLE: RESOLUTION OF THE PARADOX

(Forward, 1998; 1997) proposed to extract energy by starting with a cavity with an initial state $(a_2/a_1, a_3/a_1) = (1, 3.3)$, which is on the zero energy curve of Figure 1. Presumably the force on the walls is zero since the average energy density is zero. Then a_3/a_1 is held constant while a_2/a_1 is increased quasistatically, giving a positive energy density. If we assume the energy density is isotropic (which is not correct), then the side would experience an outward force, and expand quasistatically, doing work, until a_2/a_1 reached the zero energy curve at $a_2/a_1 = 1.8$. If we assume isotropic energy density (which is not correct), then the force would be zero at this point and the expansion would end. The system is then returned to its initial state by moving along the zero energy line. (Forward, 1998) states that no work is done when moving the state of the system along the zero energy line since the force on each face is zero during this final transformation. Although the forces are not zero because the energy density is not isotropic, the conclusion, that no energy is used, is correct since there is no net change in energy during this transformation, and no net work is done. Thus in the proposed cycle, based on the assumptions made, energy is extracted from the vacuum and no net work is done. However, because of the incorrect assumptions, net work is actually done when extracting energy from the vacuum and energy is conserved as expected.

In the first part of the cycle described by (Forward, 1998) , one of the unit dimensions in a 1 x 1 x 3.3 cavity is increased to about 1.8. By the calculations shown in Figure 2 (Hacyan, 1993), <u>initially</u> the pressure on this side is negative in spite of the fact that the total energy density is positive ($C = 3.3$, $P_1 < 0$). Thus work is being done to increase this dimension. The calculations have <u>not</u> been done to show how this force changes as the displacement increases. But it is probable that by the time the endpoint of 1.85 is reached, and a 1 x 3.3 x 1.85 cavity is obtained, the sign of the force has probably changed, so that the net work to go from $a_2/a_1 = 1$ to $a_2/a_1 = 1.85$ will be zero. On the return along the zero energy curve, there will be no net work done. This is consistent with the observation that the pressure P_3 on the 1 x 1.8 face changes sign as we go along the zero energy curve from $a_2/a_1 = 1.85$ to $a_2/a_1 = 1$. P_3 changes sign when the derivative of the energy with respect to a_3/a_1 vanishes, which is at about $a_2/a_1 = 1.7$. Until additional calculations are done, we do not know precisely how the energy and force change during the first expansion when a_2/a_1 increases from 1 to 1.85 while a_3/a_1 remains constant. We have considered other cycles for which computations are available, and all showed that the energy extracted from the vacuum equaled the work done.

OSCILLATIONS OF WALLS IN RECTANGULAR CAVITIES AND OTHER PHENOMENA

There are a number of unusual properties of rectangular cavities in addition to those already discussed. The force on C for the 1 x 1 x C cavity is approximately constant for $C>1$. If we vary C periodically about $C=1.8$ in a quasistatic manner (see Figure 2), no net work is done. At $C=1.8$, the pressure P_1 on the other faces is zero. As C is increased, the pressure P_1 will become negative or attractive. Causing the opposing 1 x 1.8 walls to approach each other. Conversely, as C is decreased, the pressure P_1 will become positive, causing the wall to tend to move apart. Thus the periodic motion of C (causing the 1 x 1 faces to oscillate) will result in a periodic variation in the pressure P_1 on the four 1 x 1.8 faces. The cavity tends to act as if it held a fluid.

This phenomenon indicates a high degree of coupling of modes in orthogonal directions. This interesting motion suggests that we may be organizing the random fluctuation of the EM field in such a way that changes in pressure directly result, which could lead to work being done. One interesting question is can we design a cavity that will just oscillate by itself in the vacuum. One approach to this would require a set of cavity dimensions such that the force on a particular side is zero, but if the side is moved inward, a restoring force would be created that would tend to push it outward, and vice versa. Hence a condition for oscillation would be obtained. Ideally, one would try to choose a mechanical resonance condition that would match the vacuum force resonance frequency. More complex patterns of oscillation might also be possible. The cavity resonator might be used to convert vacuum fluctuation energy into kinetic energy or thermal energy. More calculations of forces within cavities are needed to determine if this is possible, what would be a suitable geometry and how the energy balance would be obtained.

Only a few geometries have been examined with regard to the Casimir energy densities and forces. We are at the infancy of our understanding of such systems. Geometrical structures might be used as mechanical systems in themselves or to prepare atomic beams by passing atoms through regions in which the vacuum energy has been purposely altered, for example, to control spontaneous emission, or alter energy levels. There is evidence that the index of refraction is different from 1 in regions in which the Casimir energy is non-zero (Barton, 1997; Maclay,

1978). Biological structures may also interact with the vacuum field. It seems possible that cells, and components of cells, for example, the endoplasmic reticulum may interact with the vacuum field in specific ways. A cell membrane, with a controllable ionic permeability, might change shape in such a way that vacuum energy is transferred. Microtubules, in cell cytoskeletons, may have certain specific properties with regard to the vacuum field. Diatoms, with their ornate geometrical structures, must create interesting vacuum field densities; one wonders if there is a function for such fields.

CONCLUSIONS

Consideration of the details of the vacuum forces exerted on perfectly conducting rectangular cavities has revealed some unexpected results, for example, that an average positive (negative) energy density in the cavity does not necessarily mean the forces are outward (inward). This property has been used to resolve the paradox of extracting energy from the vacuum without doing any work. It is clear that there are very interesting features in these cavities that we do not understand, but that may prove useful in engineered devices and systems. Study of this system makes us realize how little we do understand in general of the behavior of vacuum fluctuations in regions where there are complex surfaces and what rich physics awaits us.

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