

FEASIBILITY OF COMMUNICATIONS USING QUANTUM ENTANGLEMENT

NIAC Phase 1 Final Report

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DISCLAIMER: Communication via entangled states is a controversial subject. Participation by consultants or other individuals in this research does not imply they believe it is necessarily possible, but at least they believe it is worthy of serious investigation.

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1 Summary of Key Ideas of Phase 1 Research

For all space missions, it is imperative to have reliable communication links to transmit data, computer codes, or other information. The current electromagnetic communications technologies (including laser, RF, X band, S band) do not scale well as the mission distance increases. With current methods, the power, weight, cost and complexity increase rapidly with distance, while the transmission reliability decreases. We propose to explore the possibility of a revolutionary approach to communications based on recent theoretical and experimental developments in quantum physics, in particular based on quantum correlations between entangled atoms or ions (EPR pairs). Recent experiments have verified the existence of quantum correlations between entangled photons, in which the polarization measurement of one photon is always correlated with the measured polarization of another, distant photon[13][30][55][80]. Similar results have been reported for

entangled ions[81]. Theory indicates it is not possible to use entangled pairs for a communication system if one employs standard quantum mechanical measurements to extract the information. However, if newly proposed methods of securing the information are employed, such as “protective measurements,” in which the wave function does not collapse[4][6], it appears there may be a possibility of transmitting information by non-local means. Protective measurements, which are based on the standard quantum formalism, have very different features from the usual von Neumann quantum mechanical measurements and have yet to be verified by experiment.

There are three features that characterize a standard quantum mechanical measurement[82] [67] of an observable of a quantum system:

1. the system becomes entangled with the measurement apparatus,
2. the wave function describing the entangled system-apparatus collapses,
3. the result of the measurement corresponds to one of the possible eigenvalues of the operator that was measured.

In a protective measurement, the system evolves continuously in time, according to an adiabatic interaction Hamiltonian[77]. Before describing the protective measurement further, it might be helpful to clarify what we mean by an “adiabatic perturbation” since this is a concept which is central to our research effort. An adiabatic perturbation is one that is applied very slowly with respect to the characteristic times of the system. Consider, for example, slowly moving the equilibrium point of a harmonically oscillating particle. If the oscillator is initially in the ground state, to first order it will remain in the ground state provided the movement of the equilibrium point is slower than the maximum classical velocity of the particle. There is a finite probability that the system will be in the first excited state. However, this probability is small and it varies sinusoidally in time; it does not increase monotonically with time. Some characteristics of the harmonic oscillator wave function are “adiabatically invariant” during this perturbation process, such as the number of nodes[60]. Consider another adiabatic perturbation in which an atom is placed in a slowly varying electric field $E(t)$, which is turned on slowly, giving the atom the time to adjust to the field without changing quantum numbers. In a semi-classical picture, the electron makes many revolutions during the turn-on time so the atom adapts to the perturbation without a transition to another state. The instantaneous wavefunction of the atom subject to an adiabatic perturbation can be described in terms of the eigenfunctions of the instantaneous Hamiltonian. As the perturbation is applied, the instantaneous energy may change, but it must not cross the energy level of another state, otherwise a transition to that state may occur[61].

In the quantum theory of measurement, the system interacts with a “pointer” which is the source of the classical information. A “protective” measurement on a quantum system is an extension of the notion of an adiabatic perturbation. A extensive and recent review of “protective” measurements concludes (based on theoretical analysis) that it is possible to measure a state without altering it providing the coupling to the measurement system is weak and the measurement is adiabatic[33]. The adiabatic “protective” measurement is designed to avoid entanglement between the measured state and the quantum probe, and thereby leave the measured state unchanged for all measured variables. We would hope that “protective” measurements would therefore leave the entanglement of the EPR pairs intact, but this critical question needs to be verified experimentally. **During the protective measurement, the state is characterized by a unitary evolution in time so there is no collapse of the wave function. In addition the system does not become entangled with the apparatus. As a consequence of these properties, the result of a protective measurement on a state is the expectation value of the operator for that particular state, not an eigenvalue of the operator** [11]. Using protective measurement it may be possible to send information using the mechanism of quantum mechanical correlations of entangled states. **Our approach is to adiabatically perturb one member of an EPR pair, and observe the correlations in the movements of the distant entangled member. Since the perturbations are adiabatic and the wavefunction never collapses, the communications channel should, in principle, remain intact and provide a continuously operating communications link. Nevertheless it is possible that fundamental laws of physics may prevent this communication from occurring.**

If experiment verified that the proposed approach for communications using EPR pairs was viable, it should be possible to develop an almost ideal communication system because the communication link is non-local. By non-local, we mean that there is no known mechanism by which a signal is transmitted from one particle to another, instead the members of the EPR pair seem to be linked as if they were interacting with no distance between them. Einstein referred to this as a “ghost like interaction”[40]. No power is required

to “broadcast” a signal and no signal has to pass through the atmosphere or space. Because of this non-locality of the EPR interaction, no broadcast power supply or antenna is required, no environmental noise or interference is present, and the signal does not fall off as the inverse square of the distance. Further since the link is due to entanglement between the two members of an EPR pair, the link should provide absolute security. The data rates are determined by the characteristic frequencies of the systems. For entangled ions, these frequencies are those of atomic motion, and consequently high data rates should be possible. Further, there have been great advances in the manipulation of entangled ions primarily driven by the research for quantum computing[53][83]. The methods developed for manipulating entangled ions suggest that there should be no fundamental difficulty in constructing a compact, and light weight system for communications should the proposed method be validated experimentally.

2 Objectives of Phase I Research

The primary objectives of the Phase I effort were to

1. Begin an investigate the theoretical possibility of communication using entangled states.
2. Develop a model of a quantum system, showing the behavior of the members of an EPR pair, in which the members can be separated, one member can be adiabatically perturbed and we observe the other member. Consider the measurement issue. (By an EPR pair we mean an entangled state of two particles described by the spin wavefunction $\phi = \frac{1}{\sqrt{2}}(|+\rangle_1|-\rangle_2) - |-\rangle_1|+\rangle_2$) . The total spin is 0 for this singlet state[1].)
3. Work with experimentalists to begin to develop a plan for an experiment that would test the basic concept of communication via entangled states, at least an aspect of it.
4. Develop a preliminary physical communications architecture.

3 Summary of Results of Phase I Research

All the objectives of Phase I were met, and these results will be discussed in the following sections. We will review the basic theory about communication using entangled states based on our work and that of our consultants. The encouraging results of model calculations based on the “causal interpretation ” of quantum theory will be presented. In addition, we present the results from a model using standard quantum mechanics to describe an entangled system. We interacted with two major experimental groups (Dr. David Wineland’s Ion Storage Group at NIST in Boulder, and Prof. Edward Fry’s group at Texas A and M) developing preliminary ideas about possible experiments. We considered the potential characteristics of a communications architecture and communications protocol assuming communications via entangled states was indeed possible, and it appears technical implementation would not be a major difficulty.

The Phase I effort has an encouraging conclusion: No results in our Phase 1 effort have convinced us that communication via entangled states is clearly *impossible*. Conversely no results have convinced us that it is clearly *possible*. We found that little research has been done studying the interaction between components of an entangled system. Most of the effort has been in two areas: 1) Entanglement interpreted as action-at-a-distance in the “causal interpretation” of quantum mechanics, originated by David Bohm[19][36][32]; and 2) in the area of “impossibility proofs”, primarily showing that one cannot affect the statistics of a measured value of an observable in one region by local alterations in the entangled system in a separate region[38][43]. These impossibility proofs all assume, among other things, that there is no interaction between the components of the entangled systems[39]. John Bell, whose seminal work [14] paved the way for understanding non-locality and all the modern EPR measurements, said “..what is proved by impossibility proofs is lack of imagination[15].” We discussed these controversial questions extensively in our Phase I proposal.

Our belief is that research needs to be done that considers the nature and characteristics of the interactions between portions of an entangled system, including the case in which there is a potential energy term that causes an interaction between the components. If an interaction

is present, then it should clearly be possible to communicate between the systems, yet even this case has not been considered in the literature as far as we know. We are in the process of writing a paper on a model interacting system in ordinary quantum mechanics (discussed in Section 7) that is one of the first papers we know to begin considering the relevant issues[9].

4 Summary of Requirements for a Communication System Based on Entangled States

One of the first objectives in the theoretical analysis was to determine the requirements for a communications system based on the adiabatic perturbation of entangled ions. The detailed requirements appear to be the following:

1. The system must have entangled particles (atoms, electrons, ions, photons etc.) that can be separated without altering the entanglement.
2. It must be possible to adiabatically perturb the states of the system by applying a perturbation to the particles in one region of the system.
3. The perturbation must not end the entanglement or alter the state of the system.
4. The entangled state must be characterized by a quantum number that represents a conserved quantity, such as angular momentum in the case of a spin zero singlet state (EPR pair).
5. The perturbation must alter adiabatically the conserved quantity for one of the particles in the system to insure a correlation between the entangled particles. (For simplicity, we assume the particles are distinguishable here.)
6. A correlated adiabatic change in the conserved quantity must be present for the other entangled particle member.
7. It must be possible to detect the correlated change in the wavefunction of the entangled particle.
8. The detection of the correlated change in the wave function must not end the entanglement between the EPR pair or collapse the wavefunction.
9. The interactions (perturbation and detection) must be reversible in time.

Our approach is to address each of the requirements in turn. Items 1-5 are comparatively easy to attain. In the Phase I effort, we have found some theoretical validation for Item 6, a critical item, which is presented in Sections 6 and 7. Items 7-9 appear to be the most challenging items and are the focal point for proposed future work.

We found it convenient and useful to follow a suggestion from consultant Dr. David Wineland of NIST, and one of the world leaders in experimentation on entangled ions. He suggested casting our system in terms of spin $\frac{1}{2}$ particles and two state systems. This approach is the trend today, which is a reflection of the interest in quantum computing. Quantum computing and quantum communications (encryption) drives most of the work in entangled ions and entangled photons today[24]. On a more fundamental level, Feynmann proved a general correspondence between any two-state quantum system and an equivalent two-state spin $\frac{1}{2}$ system. Thus we will do our analysis in terms of spin $\frac{1}{2}$ particles, which greatly simplifies the analysis, and puts our communications protocol in the common language of today's experimentalists.

5 Preliminary Analysis of Our Proposed Communication Approach Using Spin $\frac{1}{2}$ Particles

The quantum protocol for the proposed communications approach should be expressed in terms of spin $\frac{1}{2}$ systems or, in the language of quantum computing, qubits[65]. The most general wave function for a spin $\frac{1}{2}$

system is a linear combination of spin up $|+\rangle$ and spin down $|-\rangle$ eigenfunctions[72]:

$$\psi = \alpha |+\rangle + \beta |-\rangle$$

where the axis of quantization is arbitrary, and customarily taken as the z-axis. We assume that ψ is normalized so $|\alpha|^2 + |\beta|^2 = 1$. This expansion follows since the $|+\rangle$ and $|-\rangle$ eigenfunctions are a complete set for the expansion of any wavefunction ψ for a spin $\frac{1}{2}$ system. When this state is rotated, the coefficients α and β are changed. If the state is rotated so the total spin axis is along the positive z-axis, then the rotated wavefunction will be just

$$\psi' = e^{i\chi} |+\rangle$$

The unitary rotation operator for an infinitesimal rotation of magnitude ϵ about the unit axis \vec{u} is

$$R_u(\epsilon) = 1 + i\epsilon \vec{s} \cdot \vec{u}$$

where the spin operator is $\vec{s} = \vec{\sigma}/2$, and $\vec{\sigma}$ represents the Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

A finite rotation is represented by the operator

$$R_u(\theta) = \mathbf{I} \cos\left(\frac{\theta}{2}\right) - i \vec{\sigma} \cdot \vec{u} \sin\left(\frac{\theta}{2}\right)$$

where \mathbf{I} is the identity operator. For example, if we rotate the wave function $|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ that represents the spin up state (in the z- direction) by an angle about the x-axis, we obtain the following superposition of spin up and down states:

$$R_x(\theta) |+\rangle = \cos\left(\frac{\theta}{2}\right) |+\rangle - i \sin\left(\frac{\theta}{2}\right) |-\rangle$$

Similarly any adiabatic perturbation, which must preserve the norm, applied to the state $|+\rangle$ can be represented by the above linear combination with some value for θ . Thus the problem of measuring the adiabatic perturbation of the spin state of a single particle can be reduced to the problem of determining the two coefficients. The advantage of using a two-state system is that we need determine only two complex coefficients. For the two-state system, an adiabatic perturbation is the same as a rotation.

Since the two coefficients are complex, there are four unknowns. The normalization of the wavefunction to unity effectively eliminates one unknown, and there is an overall undetermined phase to the wave function. Hence there are two unknown, which physically can be interpreted as specifying the direction (θ, ϕ) of the spin $\frac{1}{2}$ vector, and therefore two measurements are required.

In our experiment, we anticipate having an EPR pair corresponding to an entangled singlet state (total spin $S = 0, S_Z = 0$)

$$\phi = \frac{1}{\sqrt{2}}(|+\rangle_1 |-\rangle_2) - |-\rangle_1 |+\rangle_2$$

where the axis of quantization may be taken as the z- direction. The subscript 1 refers to particle 1 and the subscript 2 refers to particle 2. **This state has no total angular momentum ($S = 0$) and is therefore spherically symmetric. This wavefunction describes a state in which the spins of the two particles are entangled.** By entanglement, we mean it is not possible to express the wavefunction for the two particles as a product of a wavefunction for particle 1 times a wavefunction for particle 2[1]. On the other hand, the wavefunction describing two non-interacting particles is typically written as a product of wavefunctions for each separate particle. **The fact that the singlet wavefunction is entangled means that it is NOT possible to discuss particle 1 as being “separate” from particle 2 since they are both components in the same quantum mechanical system[35].**

Use of the Classic Stern-Gerlach Method to Measure Spin Correlations of an EPR Pair

One of the classic experiments to measure spin is the Stern-Gerlach experiment, in which a particle with a magnetic dipole moment $\mu\vec{\sigma}$ passes through an inhomogeneous magnetic field[22]. The interaction Hamiltonian is

$$H_I = \mu\vec{\sigma} \cdot \vec{B}$$

The inhomogeneous field \vec{B} couples to the dipole moment, causing the spin to produce a force on the particle:

$$\vec{F} = -\mu\vec{\sigma} \cdot \nabla\vec{B}$$

Since the force depends on the spin, the deflection of the particle depends on the spin. Thus the Stern-Gerlach analyzer separates a beam of unpolarized particles into a spin up portion and a spin down portion.

The correlations in spin for a EPR pair can be determined using two Stern-Gerlach analyzers. One particle passes through one analyzer, and the other particle through another analyzer, with one B field possible at an angle θ to the other B field. If the B fields are both in the same direction, then whenever particle A is seen to be in a spin up state, particle B will be in a spin down state, and vice versa so in all cases the total spin in the z- direction is zero.

Note the usual formulation of quantum mechanics does not describe the measurement process in any detail. This is because the measurement process, unlike all other processes in the quantum mechanics, is NOT a unitary evolution of the state described by the Hamiltonian[35]. In the usual Copenhagen interpretation of quantum mechanics, one simply says that the wavefunction has “ collapsed” to one of the components in the wavefunction. For the spin measurement of the entangled singlet one says that, for example, one measured spin $-\frac{1}{2}\hbar$ for particle 1. From the structure of the wavefunction, this means the wavefunction has collapsed to the $|-\rangle_1|+\rangle_2$ term, so we must obtain $+\frac{1}{2}\hbar$ for particle 2. It is because of this structure of the wavefunction that the measured spins are always correlated:

$$\phi \Rightarrow \text{collapses with measurement} \Rightarrow |-\rangle_1|+\rangle_2 \Rightarrow S_{Z_1} = -1/2; S_{Z_2} = +1/2$$

On the other hand, if a spin of $+\frac{1}{2}\hbar$ had been measured for particle 1, then the wavefunction would have collapsed to the first term in ϕ , namely $|+\rangle_1|-\rangle_2$. The quantum theory of measurement says that the probability of collapse to a component is proportional to the square of the absolute value of its coefficient. It is important to note that because of the structure of the wavefunction, the correlation in spin measurements for particles 1 and 2 is completely independent of the spatial separation of the particles. Thus no matter how far particle 1 is from particle 2, the same spin correlation is present. As a consequence, any signalling that might be possible via entanglement should not be effected by the separation between the particles.

Role of Adiabatic Perturbations

If we rotate or adiabatically perturb[61] particle 1 and particle 2 in the singlet state in exactly the same way, or conversely rotate the entire measuring apparatus, then the state changes only in its overall phase. This singlet state is rotationally invariant (up to a phase factor) since it corresponds to a state with zero total spin $S=0$. The total spin the z-direction $S_{Z_1} + S_{Z_2}$ is clearly always 0. On the other hand, we can rotate (or perturb) one particle and not the other particle if we assume the particles are distinguishable. In terms of Wineland’s experiments, this is implemented by applying an electromagnetic field from a laser to either particle 1 or to particle 2. If we rotate 1 about the x-axis by an angle θ , for example, we obtain a new state

$$\phi' = \frac{1}{\sqrt{2}}[\cos(\frac{\theta}{2})(|+\rangle_1|-\rangle_2 - |-\rangle_1|+\rangle_2) + i\sin(\frac{\theta}{2})(|+\rangle_1|+\rangle_2 - |-\rangle_1|-\rangle_2)]$$

Each of the four components of this wavefunction gives the probability for the corresponding measured values of the spins of particles 1 and 2. In our proposed gedanken experiment, we might perform an adiabatic rotation of this type on particle 1. Our goal then would be to determine the coefficients of the different components of the rotated wavefunction without collapsing the wavefunction. If the components were the same as those given in the above equation for ϕ' , then we would know that particle 1 rotated but

that particle 2 did not rotate in a corresponding way to particle 1. On the other hand, if the rotation of particle 1 induced a correlated rotation in particle 2, then the resulting wavefunction would be expected to be the same (within phase factors) as the original ϕ . Thus a knowledge of the coefficients of a rotated wavefunction can tell if the perturbations applied to particle 1 were automatically mirrored in the motion of particle 2. If this correlation between the adiabatic perturbations in the spin states of particles 1 and 2 were in fact observed, then we would have the distinct possibility for communications using entangled states.

The Measurement Problem

Assuming that indeed the particles remain continuously entangled (theoretical evidence for this follows in Section 6), then **the most difficult requirement for communication using entangled states is the determination of the coefficients in the rotated wavefunction which must be done without collapsing the wavefunction and ending the entanglement between the two ions.** If a typical quantum mechanical measurement were employed two fatal problems would arise: 1) to secure the needed information from the wavefunction would require at least two measurements, but if the first measurement collapsed the wavefunction, the second measurement would be on the collapsed state, which would be a different state from the original state, and therefore no useful information would be obtained; 2) collapse of the state would end the entanglement which would terminate the communication link.

These two stringent requirements on securing information about the entangled state imply that a typical quantum mechanical measurement cannot be employed. In a typical quantum mechanical measurement, the measuring apparatus becomes entangled with the system, which is followed by the non-deterministic non-unitary collapse of the wavefunction. As we discussed previously, in our attempt to avoid the collapse of the wavefunction, we are considering using a new type of measurement, called a **protective measurement**, in which the state does not collapse[4].

Protective Measurements

The purpose of a protective measurement is to secure information about a quantum mechanical state without collapsing the wavefunction. The “protective measurement” scheme is an innovative and relatively new idea for performing a non-collapse “observation” on a quantum system to determine the expectation value of a measurement operator on the quantum state of interest without modification of the state[11]. This is in contrast to a conventional von Neumann quantum measurement, which collapses the wave function of the state into an eigenstate of the measurement operator[82]. There are several ways in which a protective measurement may be conducted[33]. One approach for this new type of quantum observation is to apply the measurement operation at very low strength over a very long time period (compared to a characteristic time of the system) in order that the state of the system evolves adiabatically, and remains an eigenstate of the instantaneous Hamiltonian[61]. Since the system remains an eigenstate, it does not become entangled with the apparatus, and it does not decohere and collapse as in a conventional quantum mechanical measurement. In contrast, with a conventional measurement, the measurement operation is applied very strongly for a very short period of time. The area of protective measurements is new, and yet to be fully explored. To date, one specific experiment has been proposed[66] to validate the theory of protective measurements, but no experiments have been conducted to date.

The “protective measurement” scheme appears to make possible determination of the quantum mechanical state of a system in a way that had previously been thought to be impossible. In particular, it appears to allow repeated protective observations of expectation values using as many alternate measurement bases as desired. In order to make a protective measurement, theory indicates certain restrictive conditions must be met:

1. The eigenstate of the system must be a non-degenerate state. If it is degenerate then transitions to these states may occur.
2. The interaction with the pointer of the measurement apparatus must have a suitable form so that the interaction is an adiabatic perturbation of the state.
3. The interaction must result in changes in the wavefunction that provide information about the state.

The example that is discussed in the theoretical papers is that of a Stern-Gerlach spin measurement. This approach would have to be modified to make a protective measurement in the confines of an ion trap.

6 Theoretical Model of Behavior of Entangled States using Causal Interpretation of Quantum Theory

In our Phase I investigation, we considered several theoretical approaches to investigate the feasibility of communication via entangled states. Using models of EPR pairs and the causal interpretation of quantum theory, it can be show that continuous correlations in spin are indeed predicted for EPR pairs subjected to magnetic fields. As discussed above, existence of these correlations is a requisite for communications via quantum entanglement.

The causal interpretation of quantum mechanics pioneered by David Bohm is very attractive for our purposes since it allows one to compute a quantum mechanical trajectory for a state. The theory describes the continuous causal evolution in time of the operators corresponding to observables, such as spin. A unique feature of the Bohm interpretation is that the observables evolve continuously to the measured value, which is the same value as predicted by standard quantum mechanics[51]. This viewpoint is very useful since our approach requires the continuous evolution of systems when subjected to adiabatic perturbations. The results based on the causal interpretation should be believable since for non-relativistic quantum mechanics, the causal interpretation necessarily gives the same predictions for measurements as the ordinary interpretation[20]. This is why the causal theory is often referred to as an “interpretation” rather than a new theory. From Bohm’s perspective, the question of signalling via entanglement is meaningless since “the relativistic notion of a signal does not fit into quantum mechanics[18].” He maintains a signal must be sent between distinct, separate, autonomous particles but the members of an EPR pair are not separate particles so the idea of a relativistic signal is not applicable.

There are two very interesting features in the causal interpretation of quantum mechanics that relate specifically to non-locality such as entanglement[51]:

1. The force on a quantum particle is given by

$$m\left(\frac{d^2x}{dt^2}\right) = -\nabla V - \nabla Q$$

where V is the classical potential and Q is referred to as the “**quantum potential,**” which is defined by

$$Q = -\left(\frac{\hbar^2}{2m}\right)\left(\frac{\nabla^2|\psi|^2}{|\psi|^2}\right)$$

2. The equation of motion for the spin includes a non-local **quantum torque** T. It follows directly from the Pauli equation, that

$$d\vec{s}/dt = \vec{T} + (2\mu/\hbar)\vec{B} \times \vec{s}$$

where the quantum torque T is defined in terms of derivatives of \vec{s} and $|\psi|^2$. **These non-local quantities V and \vec{T} can affect the motion of the quantum particle at a distance.**

Analysis of Stern Gerlach Measurements of an EPR Pair using Causal Interpretation

There is a very interesting and important calculation of an entangled singlet state using the causal interpretation of quantum mechanics[37]. First a computation was done of the behavior of both members of the singlet when the spins of both particles were measured using Stern-Gerlach analyzers. When both of their magnetic fields are in the z direction, the spin in the z direction for particle 1 and for particle 2, S_{Z_1} and S_{Z_2} , respectively, both had the initial value 0 and then evolved continuously to their final values as each particle

passed through its respective Stern Gerlach analyzer. At any time t the sum of the spins in the z-direction $S_{Z_1} + S_{Z_2}$ was identically zero. This last equation is the theoretical demonstration that the spins of the two entangled particles are continuously correlated, and not just correlated at the time of a particular quantum mechanical measurement. One of the spins evolves to the final measured value of $+1/2$ and the other to $-1/2$. The spins are measured by observing the trajectory of each particle as it passes through the magnetic field. The particle is deflected up or down depending on the sign of the spin. The result is that the beam is split into two beams, with spin of $+1/2$ and $-1/2$, in complete agreement with all experiments. The interesting feature of the Bohm interpretation is that it shows the spins are continuously correlated during the entire experiment.

The second calculation using the causal interpretation of quantum mechanics considered the case in which only ONE particle was passed through a Stern-Gerlach analyzer with its magnetic field in the x-direction[37]. In the simulation of this experiment, the spins again showed the same continuous non-local correlation. One of the spins evolves to the final value $+1/2$ and the other to the value $-1/2$. The trajectory of the particle that entered the magnetic field to measure its spin showed a deflection while the particle that did not pass through a magnetic field did not show a deflection.

We developed a computer model of this interaction and the graphical results of our calculations (done by Dr. Asling of the University of New Mexico) are shown in the Figures 1-4. The deflection of particle 1 due to the impulse from the inhomogeneous magnetic field, we call x . Particle 2 did not pass through a magnetic field. We measure the deflection y of particle 2 in a direction 45° from the x axis. (This angle is present to allow us to compare to the case in which we have two Stern-Gerlach analyzers at an angle with respect to each other). It is most convenient to analyze the system in terms of spin eigenstates in the non-orthogonal directions x and y . The final spin state can be written as a linear combination of the four states $|+\rangle_1 |+\rangle'_2, |+\rangle_1 |-\rangle'_2, |-\rangle_1 |+\rangle'_2, |-\rangle_1 |-\rangle'_2$, where $\sigma_{y2} |\pm\rangle'_2 = \pm |\pm\rangle'_2$ and $\sigma_{x1} |\pm\rangle'_1 = \pm |\pm\rangle'_1$. Each pair of states is multiplied by a corresponding spatial function, for example $g_{++}(x, y) |+\rangle_1 |+\rangle'_2$. We assume the spatial part of the initial wavefunction is given by Gaussian wavepackets.

Figure 1 shows the square of the wavefunction $|\psi|^2$ on the vertical axis, describing two wavepackets corresponding to the splitting of the wavefunction in the magnetic field after a certain time. Both packets have approximately the same values of x since it is the undeflected coordinate, and the two values of y correspond to the splitting.

In Figure 2 we plot the spin of particle 1, which passed through the magnetic field, multiplied by the corresponding spatial wave function on the vertical axis, as a function of x and y . The two non-zero regions correspond to positive and negative spin, and are separated in y because of the magnetic field.

Figure 3 shows the corresponding plot for particle 2, which did not pass through the inhomogeneous magnetic field. Again the vertical axis is the spin multiplied by the corresponding spatial part of the wavefunction. The striking feature is that the spin of particle 2 is correlated with that of particle 1. From the figures, one cannot tell which particle passed through the magnetic field.

This correlation is mathematically demonstrated in Figure 4, in which we plot the sum $S_{1x} + S_{2y} \cos 45^\circ = \epsilon$ times the corresponding spatial wavefunctions for all values of x and y . This should vanish if the correlations in spin for the two particles is perfect. The largest deviation of ϵ from zero is about 5×10^{-17} , which corresponds to our computational error. **Thus the causal theory predicts that the spins are perfectly correlated if only one particle passes through a Stern-Gerlach analyzer.** Based on the results of his calculation for this example, Holland concludes[51]:

The correlations between these quantities are due to the nonlocal character of the quantum potential and the quantum torque. A key point is that the particles carry with them well defined properties from the source, just as classical particles do; these do not come into being only through discontinuous projections of “potentialities” when measurements occur [i.e. standard quantum mechanical measurements]. On the other hand, **the magnitudes of one particle’s properties may be steered into certain values solely by distant actions on the other coupled particle even though the systems do not classically interact.** The subtlety of these correlations cannot be explained by any classical force.

Holland’s words are strikingly reminiscent of those of Erwin Schrodinger, one of the founders of quantum theory almost 60 years earlier. Schrodinger coined the term “entanglement” to describe the EPR pairs[74],

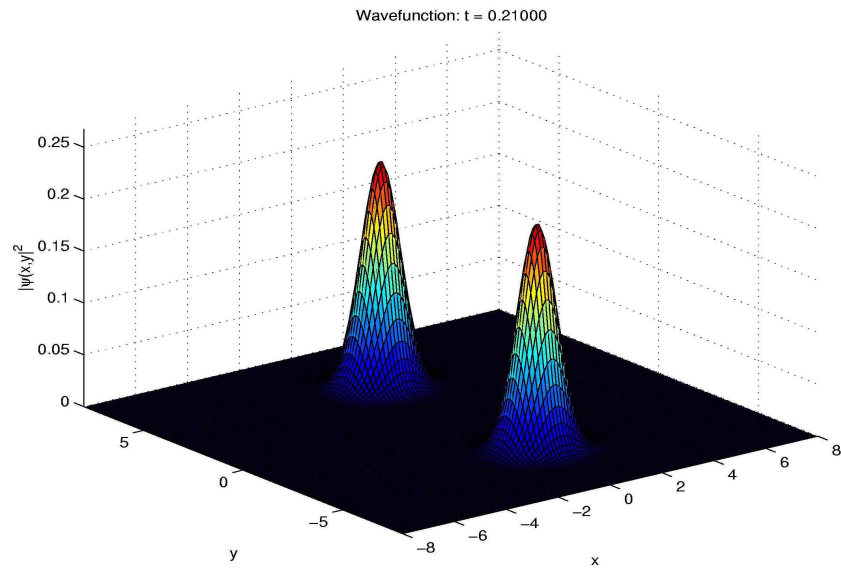


Figure 1: The wavepacket corresponding to an entangled singlet splits into two wavepackets due to an inhomogeneous field.

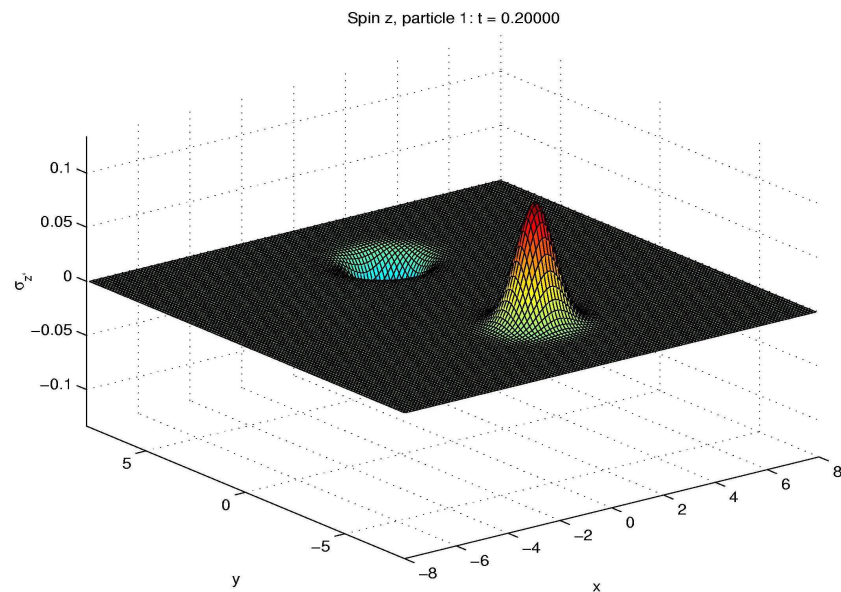


Figure 2: Plot of the spin multiplied by the spatial wavefunction squared for particle 1 which passed through a inhomogeneous magnetic field..

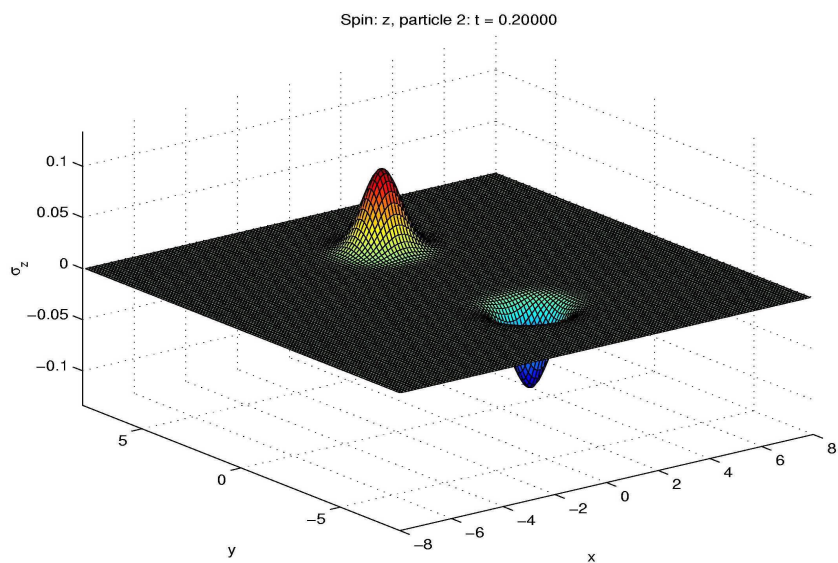


Figure 3: Plot of the spin times the spatial part of the wavefunction squared for particle 2, which did NOT pass through a magnetic field.

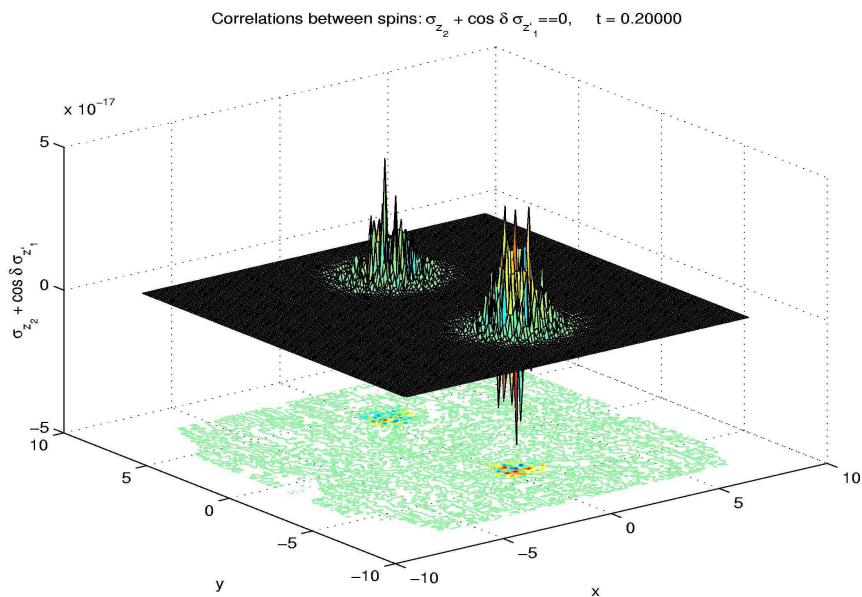


Figure 4: Plot of the sum of the spins times the spatial wavefunction squared for the two particles, which should vanish if the spin correlations are perfect. Note the vertical scale is 10^{-17} so the correlations are perfect within the calculational error.

and demonstrated that a general relationship existed between the EPR pairs, in which a “**sophisticated experimenter can, by a suitable device which does not involve measuring non-commuting variables, produce a non-vanishing probability of driving the system into any state he chooses**” [75]. Schrodinger showed that any function of the canonical position and momenta would have the same value for both members of an entangled state. Schrodinger also maintained that the entanglement between a system and a measuring apparatus is central to the quantum mechanical theory of measurement, which is at the heart of the quantum/classical boundary[76] . Holland goes on to say[51]:

The state of particle 2 [the unmeasured particle] has therefore changed as a result of a distant interaction undergone by another particle, **both particles being guided by an entangled state**. A demon sitting on particle 2 receives an instruction to start the ball rotating. It would thus appear that, at the level of their individual processes that it accounts for, **the causal interpretation implies a form of ‘signalling’, via the quantum torque**.

For completeness, he goes on to say that it is **not** consistent with quantum mechanics to use this interaction to send information since there is **no “internal standard” by which to measure the induced changes**. Using the conventional methods of quantum mechanics, we clearly agree with this statement. However, with novel methods of imparting information, such as adiabatic perturbations, and novel methods of extracting information, such as using protective measurements, the situation becomes much more complex and much less clear.

The existence of spin correlations is a very important result for our approach since it shows that the spins of entangled particles are **continuously correlated** if one spin is rotated adiabatically by a magnetic field. Thus if we were to apply an adiabatic perturbation to the spin of an entangled particle, we would anticipate that the spin of the other particle would be continuously correlated, which is requirement #6 in our list of requirements.

It is very encouraging that our calculation and the calculation of [37] give theoretical support to one of the most important requirements for the viability of communications via entangled states: if the spin of one member of an EPR pair is perturbed, then the spin of the other member is continuously correlated so $S_{Z_1} + S_{Z_2}$ remains zero. The other key requirement is the ability to observe these perturbations without collapsing the wavefunction. Our approach to this challenge is to use **protective measurements**, which we propose to investigate fully in a Phase II effort.

7 Preliminary Model of a System of Two Interacting Particles with Entangled States Using Standard Quantum Mechanics

We wanted to investigate a system of two entangled spin $\frac{1}{2}$ particles that interact through a potential that vanishes at infinity[9]. We picked an interaction corresponding to a spin-spin coupling. The primary purpose of the model is to allow us to investigate the behavior of one portion of the system when we perturb the other portion, and thereby to investigate the possibility of communication between the separate portions of the entangled system as a function of the distance between them. We take this approach because it is not entirely clear how to correctly address the case in which the distance is so great that there is no interaction between the components of the entangled system. Note that establishing the possibility of communications between separated regions requires **establishing the existence of the correlations and the possibility of measuring them**. Note that the results from a model may be model dependent. Other interactions may lead to systems with quite different properties.

We will assume that we have a pair (designated 1 and 2) of identical, uncharged, entangled spin 1/2 fermions that are moving in opposite directions along the y-axis with coordinates y_1 and y_2 . (Similar results are obtained if we assume the particles are distinguishable.) We will then apply a magnetic field in the region to the right of the origin and observe the response of the portion of the system to the left of the origin. . Because of the indistinguishability of the particles, it is not possible to describe which particle is on the left or right, instead quantum mechanics indicates there is a superposition of both. For our calculations, we made an approximation that the change in the kinetic energy during the experiment was small (impulsive approximation), so we could neglect the effect of the two kinetic energy terms in the Hamiltonian. The

Hamiltonian for our system is

$$H = \frac{P_1^2}{2m} + \frac{P_2^2}{2m} + g \frac{\vec{\sigma}_1 \cdot \vec{\sigma}_2}{|y_1 - y_2|^3}$$

where g is a constant. The $\vec{\sigma}_1 \cdot \vec{\sigma}_2$ interaction is modeled after the classical potential energy U for the interaction between two dipoles \vec{d}_1 and \vec{d}_2 located at \vec{x}_1 and \vec{x}_2

$$U = \frac{\vec{d}_1 \cdot \vec{d}_2 - 3(\vec{n} \cdot \vec{d}_1)(\vec{n} \cdot \vec{d}_2)}{|\vec{x}_1 - \vec{x}_2|^3}$$

where \vec{n} is a unit vector in the direction $(\vec{x}_1 - \vec{x}_2)$. For simplicity, we did not include the term depending on \vec{n} . Since the operator $\vec{\sigma}_1 \cdot \vec{\sigma}_2 = 2 - 3\vec{S}^2$, the eigenstates of H will also be eigenstates of the total spin, namely the usual singlet $|S\rangle$ (total spin $S = 0$, and spin in the z-direction $S_Z = 0$), and triplet ($S = 1; S_Z = -1, 0, +1$) spin eigenstates $|T_{S_Z}\rangle = |T_0\rangle, |T_1\rangle, |T_{-1}\rangle$. The spatial part of the wavefunctions is chosen so the total wavefunction is antisymmetric with respect to the interchange of particles 1 and 2. The singlet state is

$$|S\rangle = \frac{1}{\sqrt{2}}(|+\rangle_1 |-\rangle_2 - |-\rangle_1 |+\rangle_2)(\psi_L(\vec{x}_1)\psi_R(\vec{x}_2) + \psi_R(\vec{x}_1)\psi_L(\vec{x}_2))$$

and the triplet states are

$$|T_0\rangle = \frac{1}{\sqrt{2}}(|+\rangle_1 |-\rangle_2 + |-\rangle_1 |+\rangle_2)(\psi_L(\vec{x}_1)\psi_R(\vec{x}_2) - \psi_R(\vec{x}_1)\psi_L(\vec{x}_2))$$

$$|T_1\rangle = \frac{1}{\sqrt{2}}(|+\rangle_1 |+\rangle_2)(\psi_L(\vec{x}_1)\psi_R(\vec{x}_2) - \psi_R(\vec{x}_1)\psi_L(\vec{x}_2))$$

$$|T_{-1}\rangle = \frac{1}{\sqrt{2}}(|-\rangle_1 |-\rangle_2)(\psi_L(\vec{x}_1)\psi_R(\vec{x}_2) - \psi_R(\vec{x}_1)\psi_L(\vec{x}_2))$$

Here $\psi_L(\vec{x}_1)$ represents the spatial part of the wavefunction for particle 1 in the region on the left side where the magnetic field B vanishes, and $\psi_R(\vec{x}_2)$ represents the spatial part of the wavefunction for particle 2 in the region to the right which contains the B field. We will assume that the initial state is a singlet eigenstate of H :

$$|\phi(0)\rangle = \frac{1}{\sqrt{2}}(|+\rangle_1 |-\rangle_2 - |-\rangle_1 |+\rangle_2)(\psi_L(\vec{x}_1)\psi_R(\vec{x}_2) + \psi_R(\vec{x}_1)\psi_L(\vec{x}_2))$$

At time $t = 0$ we assume we turn on the interaction Hamiltonian H_I

$$H_I = -\mu(\sigma_{Z_1}B(\vec{x}_1) + \sigma_{Z_2}B(\vec{x}_2))$$

where the field vanishes for coordinates in the region on the left side, so for example, $B(\vec{x}_1)\psi_L(\vec{x}_1) = 0$. The magnetic field B is assumed to be constant in the z-direction, and to go to zero outside the region on the right side. We assume that we turn on the B field slowly (adiabatically), so that the system can adjust to the new field and therefore remains in an eigenstate of the instantaneous Hamiltonian (adiabatic theorem). In effect, the uniform B field acts a rotation operator. In terms of the basis ($|T_0\rangle, |S\rangle$), the total Hamiltonian H_T can be written as

$$H_T = H + H_I = -f \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 2f & \mu B \\ \mu B & -2f \end{pmatrix}$$

where

$$f = g \frac{1}{|y_1 - y_2|^3}$$

This form of the Hamiltonian H arises since the B field is chosen along the z-axis. (If other components of the B field were present, they would couple to the other members of the triplet. In fact, rigorously other

components of \mathbf{B} must be present if the divergence of \mathbf{B} is to vanish, as it must according to Maxwell's laws.) The total energy operator H_T can be written in terms of the Pauli σ matrices:

$$H_T = -f\mathbf{I} + \vec{a} \cdot \vec{\sigma}$$

where \mathbf{I} is the identity operator and the vector \vec{a} is $\vec{a} = (\mu B, 0, 2f)$. Define the angle θ where

$$\tan(\theta) = \frac{\mu B}{2f} = \frac{\mu B |y_1 - y_2|^3}{g}$$

or

$$\sin(\theta) = \frac{\mu B}{\sqrt{\mu^2 B^2 + 4f^2}}$$

The model can be solved, showing that with the presence of the constant magnetic field on the right side only, the singlet ground state has evolved into a linear combination of the singlet and triplet states:

$$|\phi(t)\rangle = -\sin\left(\frac{\theta}{2}\right) |T_0\rangle + \cos\left(\frac{\theta}{2}\right) |S\rangle$$

This state $|\phi(t)\rangle$ is an eigenstate of the total Hamiltonian with an associated energy $-|a| = -\sqrt{\mu^2 B^2 + 4f^2}$. The other eigenstate has energy $+a$ associated with it. Since these are eigenstates of the total Hamiltonian there is an overall time dependent phase factor, which we have omitted. The eigenstate $|\phi(t)\rangle$ has the property that, for any value of θ , the spins of the particles in the z direction are always correlated:

$$(S_{Z_1} + S_{Z_2}) |\phi(t)\rangle = 0$$

This result, which follows since $[H_T, S_{Z_1} + S_{Z_2}] = 0$, is exactly the same as that obtained when the causal interpretation of quantum mechanics was used to analyze an entangled singlet, when one particle in the singlet passed through an inhomogeneous magnetic field. It is reassuring to obtain the same basic result using the two different approaches.

One convenient way to consider the effect of the magnetic field on the portion of the system on the left side, where the magnetic field vanishes, is to compute the reduced density matrix ρ on the left side. If a protective measurement of an observable A is possible, then the result of that measurement is $\langle A \rangle$. In this model, the states are non-degenerate so a protective measurement of the density matrix should be possible. As a consequence the result of a protective measurement of an **observable A that acts only on the left side** can be written as the trace over the reduced density matrix:

$$\langle A \rangle = \text{tr}(\rho A)$$

Note that here $\langle A \rangle$ represents the result of a single protective measurement on a single system. It does not have the usual meaning of the expectation value of A , which is based on the measurement of the observable A for an ensemble. We can compute the reduced density matrix on the left side, where the magnetic field vanishes:

$$\rho = \frac{1}{4} \begin{pmatrix} 1 - \sin \theta & & & \\ & 1 + \sin \theta & & \\ & & 1 - \sin \theta & \\ & & & 1 + \sin \theta \end{pmatrix}$$

This is written with the basis $\{|+\rangle_1, |-\rangle_1, |+\rangle_2, |-\rangle_2\}$. Since the elements of the density matrix depend on θ , it is clear that changes in θ , resulting from either changes in B or $(y_1 - y_2)$, will affect the measured values of observables A . **In other words, in this model with a finite separation, if we change the value of the magnetic field on the right side, it is possible to communicate information to the region on the left side.** For example, we could measure the total spin in the z direction for both particles using local measurements on the left side only, obtaining the result

$$\langle S_{Z_1} + S_{Z_2} \rangle_{\text{LEFT}}^{\text{SIDE}} = \text{tr}(\rho [S_{Z_1} + S_{Z_2}]) = -\sin \theta$$

where the value of θ is determined by the magnetic field on the right side. The corresponding total spin in the z -direction measured on the right side is $\sin \theta$. We considered several cases in our model:

1. For a finite separation.

- (a) No magnetic field $B \Rightarrow 0$, $\sin(\theta) \Rightarrow 0$, so $\sin(\frac{\theta}{2}) = 0$ and $|\phi\rangle_{B=0} \Rightarrow |S\rangle$, which is the initial state before the magnetic field was applied. A protective measurement of the total spin in the z direction would give:

$$\langle S_{Z_1} + S_{Z_2} \rangle_{\substack{LEFT \\ SIDE}} = 0$$

- (b) Finite magnetic field B , so $|\phi\rangle_B \Rightarrow -\sin(\frac{\theta}{2})|T\rangle + \cos(\frac{\theta}{2})|S\rangle$. $\therefore \phi$ The protective measurement of the total spin in the z direction would yield:

$$\langle S_{Z_1} + S_{Z_2} \rangle_{\substack{LEFT \\ SIDE}} = -\sin \theta$$

Thus by changing the magnetic field in the right region, we can clearly communicate to the region on the left, provided the separation is finite.

2. For an infinite separation.

- (a) For any finite magnetic field at an infinite separation, $f \Rightarrow 0$, $\sin(\theta) \Rightarrow 1$ so $\sin(\frac{\theta}{2}) = \frac{1}{\sqrt{2}}$ and $|\phi\rangle \Rightarrow -\frac{1}{\sqrt{2}}|T\rangle + \frac{1}{\sqrt{2}}|S\rangle$. The protective measurement of the total spin in the z direction would give:

$$\langle S_{Z_1} + S_{Z_2} \rangle_{\substack{LEFT \\ SIDE}} = -1$$

- (b) For zero magnetic field at infinite separation, $B \Rightarrow 0$, $\sin(\theta) \Rightarrow 0$, so $\sin(\frac{\theta}{2}) = 0$ and $|\phi\rangle_{B=0} \Rightarrow |S\rangle$, which is the initial state before the magnetic field was applied. The protective measurement of the total spin in the z direction would give:

$$\langle S_{Z_1} + S_{Z_2} \rangle_{\substack{LEFT \\ SIDE}} = 0$$

For this model we should be able to perform a protective measurement of the reduced density matrix ρ and obtain these expectation values using a Stern-Gerlach analyzer since the eigenstates are non-degenerate. Without making protective measurements, the statistics of an ensemble would make any communication impossible in all cases.

The last two results, 2a and 2b, seem to imply the possibility of communicating between entangled regions at infinite separation, however, there are some additional complexities that must be considered:

1. **The limiting procedure needs to be defined more carefully: the limits of $B \Rightarrow 0$ and $f \Rightarrow 0$ may not commute with the possible consequence that signalling is not possible in this model.**
2. **Based on the analysis presented in 2a and 2b, one limitation in the signaling between the two regions of space in this model lies in the requirement that we must have an adiabatic perturbation or transitions will occur between states.** When the magnetic field is turned on, it must be done slowly enough so that no transitions are induced between the initial state and other states. In the model, the two states are superpositions of the singlet and the triplet. The splitting in energy between the two eigenstates is just $2a$, which reduces to $2f$ when B is zero. When the separation is very large, and B is zero, the splitting $2f$ goes to zero as $|y_1 - y_2| \rightarrow 0$, which means that the magnetic field must be turned on infinitely slowly or transitions will be induced to the higher energy eigenstate. **Thus, although, some form of communication may be permitted, it may take forever if the distance is infinite.**
3. When $B \rightarrow 0$ and $f \rightarrow 0$ the energy levels become degenerate and a protective measurement may not be possible.

This is an *extremely* interesting result. The preliminary analysis of this model, which is based on orthodox quantum mechanics, does *not* appear to give an absolute prohibition about signaling. Rather the model suggests that great delicacy is required in the limiting procedures, and that restrictions in the ability to signal that arise, in this case, due to the nature of the energy levels in the model. The result also appears to depend on the manner in which the perturbations were applied, and the particular observable that was measured. The results obtained with this model calculation suggest exploring other approaches in which, for example, the energy difference between states does not go to zero at infinite separation. It also suggests the need for second order perturbation theory to compute the probabilities.

8 Experiments to Test the Possibility of Communications via Entangled Particles

We met with two groups of experimentalists to discuss possible experiments: Dr. Wineland, Group Leader in Ion Storage, Time and Frequency Standards Division, NIST, Boulder CO, and Prof. Edward Fry, Physics Department, Texas A & M. Both groups use the equivalent of spin $\frac{1}{2}$ systems in their experiments, generating singlet states in which the spins correspond to the nuclear orientation, which is mathematically indistinguishable from spin 1/2 states arising from paired electron spins.

Meeting with Dr. David Wineland of NIST

The PI spent 2 days at NIST in Boulder, in discussions with Dr. David Wineland, who heads one of the top experimental groups in the world dealing with entangled ions. His group recently published a cover page article in Nature describing the first case of entanglement of 4 ions[71]. The $^9\text{Be}^+$ ions are held in Penning traps, which provide the equivalent of a harmonic trapping potential. By exposing the ions to various lasers causing Raman transitions, the NIST group can couple the hyperfine state of the ion to its motional state in the harmonic well[53], and generate entangled states[63]. Their states are typically singlets or $|+\rangle_1|+\rangle_2 + e^{i\varphi}|-\rangle_1|-\rangle_2$ for two ions, and superpositions like $|+\rangle_1|+\rangle_2|+\rangle_3|+\rangle_4 + e^{i\varphi}|-\rangle_1|-\rangle_2|-\rangle_3|-\rangle_4$ for the four entangled ions.

Dr. Wineland thought an experiment exploring the non-local coupling between entangled ions, which are exposed to adiabatic perturbations, might be interesting. He had several suggestions about possible experiments. As has been discussed, one of the key challenges that must be met is extracting information about the correlated perturbation without collapsing the wavefunction. He had an interesting suggestion for addressing this pivotal issue, namely to utilize “ancilla” ions, which are weakly entangled with the “signal” ion that state of which we seek to determine (without collapsing the wavefunction)[65]. A standard measurement is made of the ancilla ions, with the hope that the coupling of the signal ion to ancilla is strong enough so that the ancilla carries information about signal ion, but weak enough so that a measurement on the ancilla will not end the EPR entanglement. Dr. Wineland outlined possible steps to an experiment:

1. Develop a quantum protocol for the exact experiment we propose.
2. Put the protocol in terms of a system of spin 1/2 particles.
3. One possible approach to an experiment would be the following:
 - (a) Entangle two ions. (This process still is challenging.)
 - (b) Separate these ions into two regions. They are beginning to master the procedure to separate ions by micron sized distances.
 - (c) Weakly entangle one of the ions, say #1, with some “ancilla” ions. This is not an uncommon procedure. The degree of entanglement is controlled by the duration of a laser pulse.
 - (d) Apply an adiabatic perturbation to ion #2, such as illumination by one or more laser beams
 - (e) Try to measure the correlated perturbations of the entangled ion #1 by observing the “ancilla” ions, which are weakly entangled with ion #1. In this manner one hopes to avoid collapsing the wavefunction. A standard measurement is made of the ancilla ions, with the hope that this measurement will not affect the primary entangled ion #1.

- (f) Dr. Wineland suggested considering various interaction that might couple the ancilla to the primary ion.

Clearly there are many complex considerations in the design of a careful experiment, such as the relationship of the energy levels, the effects of the containing potentials, the energy and polarization of the lasers etc[83]. Dr. Wineland is interested in doing an experiment if we make sufficient theoretical progress. From the theoretical viewpoint, we have yet to understand the effect of a “weak entanglement” with the ancilla and how such a measurement would affect the outcome and significance of an experiment, or if it is a suitable approach. Conversely we may need to employ some type of protective measurement. We need to determine how to do a protective measurement in the environment of the ion traps. Our belief is that we need to significantly increase our theoretical understanding before we can design and execute a careful experiment. We believe we will need a detailed model of the experiment in order to understand the results expected and to identify and understand results that are not expected.

Meeting with Dr. Edward Fry of Texas A and M

Roger Lenard visited Prof. Edward Fry at Texas A & M. Prof. Fry has been preparing a two-beam laser system to be used for a “loop-hole free test of Bell’s inequalities” that employs lasers to dissociate a dimer of mercury into two atoms that move apart[42]. Prof. Fry is interested in conducting a proof-of-principle experiment for communication via entangled states and, working with Roger Lenard, has provided us with an outline/budget of his proposed approach. In the Fry experiment ^{199}Hg dimers are generated in a supersonic jet. Using simulated Raman transitions, the dimers are dissociated in such a way that they generate entangled atoms in a singlet state[41]. The TAM group is able to generate ions from the Hg atoms, which facilitates storage in Penning traps, and experiments of the type we contemplate.

9 Development of Communication Architectures(written by Roger Lenard)

The systems of entangled ions which we are considering for potential use in a communications system are all equivalent to two state, spin 1/2 systems. Thus they are digital systems, and therefore well suited to be adapted to the digital communications protocols that are in common use today in satellite communications. It is our conjecture at this time that while the architecture of quantum-level communications may be radically different from conventional RF or laser -communications architectures, the protocols may be able to remain unchanged. This, we believe would be a significant boon to acceptance of the concept, because much of the pre-processing subsystem architecture can remain unchanged. The interface components will be quite different.

In our proposal we spent some time discussing technical aspects of how perturbations might qualify for low-power communications links without providing a representational architecture. At that time, the concept was still too immature to wrap it into an architectural framework that provided space system analysts with an inkling of how such a system might be formed. Since that time our team has learned a great deal about the foundations of adiabatic perturbations, protective measurements and entangled quantum systems. However, before we present the details of a specific concept for the architecture, it is necessary to provide a basis for understanding how present communications work.

The key requirement for communications is that a system must have at least one common referent. Typically, in radio frequency (RF) communications, this is supplied by a carrier band that operates at a certain frequency. Most space communications operate in frequency ranges from C-band to Ka band. To develop a communications system, the designer selects a given carrier frequency, which is typically a continuous wave, but which could under certain circumstances be a pulse modulated signal. A transmitter/receiver is located at one or sites on the ground, and another transmitter/receiver is on the spacecraft. Sometimes a series of frequencies are used to carry commands and another to carry data. Sometimes the up-link from ground to spacecraft is at one frequency, and the downlink is at another frequency. Regardless of the frequency selected, the reference must be common to both if communications can occur.

The CW carrier can, by itself transmit no information if it is not either pulsed or modulated in some fashion. There are many forms of modulation, but typically, the sender has a local oscillator upon which

information is coded that is combined with the carrier. The local oscillator is limited to about 20% of the carrier frequency's maximum frequency, so it may have a bandwidth of only 20%. At the receiving end, the spacecraft receiver is designed to beat the incoming waveform with the same local oscillator frequency as the transmitter. The net result is a series of signals that the information modulated waveform can decode, provided there is another common referent, that of the meaning of a given modulation. Both transmitter and receiver must know this information. For digital satellite systems today, often differential (DE-BPSK) or binary phase shift keying (BPSK) are used, or frequency shift keying (FSK)[58].

We will sketch one communication architecture for our proposed method of communication via entangled states of long-lived ions. The entanglement means that the “signal” and “receive” ions are part of the same quantum mechanical system, described by a singlet state, even though they are well separated. Entangled “signal” ions in singlet state would be exposed to a magnetic field that would cause a characteristic precession of the spin vectors at a particular frequency and a particular angle, both of which may depend on time. The entangled “receiver” ions would respond characteristically to the adiabatic perturbation of the “signal” ions due to this perturbing magnetic field. For example, $S_{Z_1} + S_{Z_2}$ or $\vec{S}_1 + \vec{S}_2$ would remain equal to zero during the perturbation. Protective measurements would be made of the state of the “receiver” ions. This state of the entangled system would serve as one of the two characteristic carrier modes in the communications protocol.

The magnetic field applied to the “signal” ions could be altered to generate another characteristic state in the system, physically corresponding to a precession of the spin vectors at a different frequency and a different angle. The digital information we want to transmit is encoded into the two states of the system. These two states of the carrier might correspond physically to different perturbed states of the singlet, or perhaps the singlet-triplet combination. The encoding of the information could be done in a variety of ways, depending on technological considerations. The applied fields could be time dependent fields. The frequencies of these fields would determine the bandwidth of the communications system[46]. **The characteristic frequencies of electronic transitions are in the range of about 10^{15} to 10^{16} Hz. The modulation of the magnetic field is typically a per cent of the carrier, so we could have considerable bandwidth in principle. As a lower limit, we can take the energy difference between the levels of the ${}^9\text{Be}^+$ ions employed by Wineland, namely 1.25 GHz[63]. Even with this, modulation of approximately a MHz would be possible. We would expect that it would be possible, at least in principle, to make protective measurements at the required rate.**

Our architecture is shown schematically in Figure 1. “System a” consists of an ensemble of distinguishable, spin 1/2 particles, “transmitting ions”, in a location where we can impose a magnetic field. If the species being modulated is electrons, the field strength will be weak, about 10 G. If the particles used are ions, the field will most likely have to be in Tesla range. We employ signal generating equipment that is standard with any frequency modulation system used in satellite systems today[46]. Instead of frequency modulating a local oscillator, we will be time-modifying the above field with a known reference modulation corresponding to frequency $\omega_1(t)$ or $\omega_2(t)$. For illustrative purposes we might denote $\omega_1(t)$ as corresponding to a 1-bit and $\omega_2(t)$ as corresponding to a 0-bit. The signal generator develops a local voltage signal $V(t)$ as in a conventional communication system, but now this signal is processed by a power amplifier so that it can provide sufficient power to the magnetic field coils or to an RF generator depending upon the desired frequency. The field coils vary the magnetic field strength, resulting in a change to the precessional behavior. With a suitable measurement protocol, our imposed modulation should be detectable since it should be in the MHz to GHz range.

In the schematic, the receiver is shown as resonant circuit detection. In practice, we anticipate using SQUID technology because of its sensitivity. At the present, we do not have any specific requirement for sensitivity, but we anticipate high sensitivity will be required. In recent ion experiments, good projective measurements are made on single ions[50], so we also anticipate that the requirements for the proposed communications system will be technologically possible. . If we can trap thousands or millions of entangled pairs, the signal-to-noise ratio should be acceptable. The optimum approach for increasing the signal to noise ratio by using additional entangled ions is yet to be determined.

The baseline architecture assumed in our proposal has changed somewhat based on the results of our research to date. We still believe that SQUIDS can be used for detection of the impressed signal on the receiving member. We believe that much stronger magnetic fields will be required at the perturbing component. The SQUIDS will be made to cover an area of several square cm. The mass of the SQUIDS will

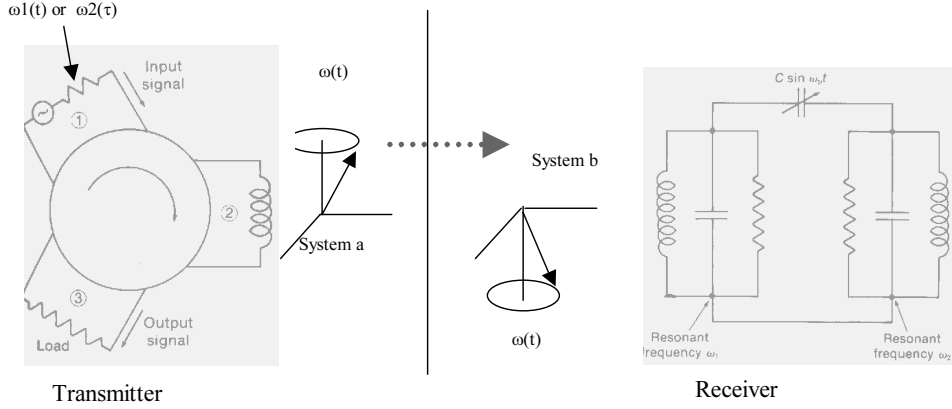


Figure 5: Schematic of quantum architecture.

be approximately 2 grams.. At 4.2 K small refrigerators have an efficiency of about 150We/W cooling. We should expect to increase this to about 200We/W for a temperature below 4.2K. For the resonant frequency modulated system, a single larger SQUID will suffice. The transmitter portion will require a field strength of 10s -1000s of Gauss. We are in the process of sizing such magnets. However, several options are available. Wire-wrapped magnets have a good ability to generate a strong and focussed magnetic field. They are well-defined and highly reliable. However, they are essentially a tuned circuit with the system resonating between the tuned inductor and capacitor. They are limited due to inductance and capacitance to about 10 MHz. Above 10 MHz, we could go to ceramic filters (such as used in cellular telephones) or tuned stubs. The question with these devices is the feasible field strength. We need to ascertain the field strength necessary to adiabatically perturb the resonant EPR system and determine if tuned stubs or ceramic filters are feasible perturbation devices. The ceramic filters operate routinely at about 900 MHz. They draw about 5 W of power in the cellular telephone mode. We may require higher power levels than a cellular telephone to result in a perturbing field of the necessary strength. Above 900 MHz it is likely that we will require either a single mode waveguide or a single mode maser to provide the resonant magnetic field of the desired field strength. **Based on our present analysis, we can form a preliminary assessment of our communications architecture. It size, weight and power are a function of modulation frequency, hence throughput and range as shown in Table 1.**

| Max Data Rate | X-Mitter | X-Mass | X-Power | X-Volume | R-Mass | R-Power |
|---------------|-----------------|--------|---------|----------------------|--------|---------|
| < 1-2 Mbps | wire coil | 100 g | 5 W | <20 cm ³ | < 2g | <0.1 W |
| <100 Mbps | stub/cer filter | 100 g | >10 W | <30 cm ³ | < 2g | <0.5 W |
| about 1 GBps | tuned maser | <10 kg | >200 W | >8000cm ³ | < 2g | < 1 W |

Table 1.Candidate Technology Assessment. In all cases the Receiver R is a SQUID device.

For most missions, the lower data would clearly suffice. Only very broad coverage, high resolution imaging camera systems would require higher data rates, as would perhaps crewed missions. For proof of principle, we will concentrate our assessment at the lower data rate range due to its greater simplicity.

10 Conclusion and Recommendation for Further Work

The Phase 1 effort has convinced us that the area of non-local interaction has received little serious attention from the physics community, other than general impossibility proofs that one cannot alter the statistics of a standard von Neumann measurement in one region by what is done locally in another separate region. With new measurement approaches in quantum mechanics, with new experimental technologies for performing

EPR experiments, it seems appropriate to investigate the nature of non-local interactions. The nature of “protective measurements” which do not collapse the wavefunction, and other measurement protocols need to be explored[6]. The predictions from the causal interpretation of quantum mechanics need to be explored as well as the predictions of orthodox quantum theory. Ultimately, an experiment will be needed to resolve the question of whether signalling between components of an entangled system is possible. The subtlety of experiments in non-locality is apparent when one considers the complications and details regarding the typical EPR experiments designed to demonstrate that quantum mechanics is inherently non-local. After 20 years, loopholes remain that plague the experimentalists[42][54]. An experiment to determine if non-local signalling is possible will be no less complicated. Before a decisive experiment can be conducted, more theory must be developed, and a detailed model of the entire measurement process is required, that would serve as a guide to designing and interpreting the results of an experiment. We propose to address these tasks in a Phase II effort.

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