

A Gedanken spacecraft that operates using the quantum vacuum (Dynamic Casimir effect)

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Version: Accepted for publication in Foundation of Physics 11/03

Conventional rockets are not a suitable technology for interstellar missions. Chemical rockets require a very large weight of propellant, travel very slowly compared to light speed, and require significant energy to maintain operation over periods of years. For example, the 722 kg Voyager spacecraft required 13,600 kg of propellant to launch and would take about 80,000 years to reach the nearest star, Proxima Centauri, about 4.3 light years away. There have been various attempts at developing ideas on which one might base a spacecraft that would permit interstellar travel, such as spacewarps. In this paper we consider another suggestion from science fiction and explore how the quantum vacuum might be utilized in the creation of a novel spacecraft. The spacecraft is based on the dynamic Casimir effect, in which electromagnetic radiation is emitted when an uncharged mirror is properly accelerated in the vacuum. The radiative reaction produces a dissipative force on the mirror that tends to resist the acceleration of the mirror. This force can be used to accelerate a spacecraft attached to the mirror. We also show that, in principle, one could obtain the power to operate the accelerated mirror in such a spacecraft using energy extracted from the quantum vacuum using the standard Casimir effect with a parallel plate geometry. Unfortunately the method as currently conceived generates a miniscule thrust, and is no more practical than a spacewarp, yet it does provide an interesting demonstration of our current understanding of the physics of the quantized electromagnetic field in vacuum.

Key Words: Casimir force, vacuum fluctuations, vacuum energy, adiabatic, spacecraft

1. INTRODUCTION

Our objective in this paper is to explore how one might make use of the properties of the quantum vacuum in the creation of a spacecraft. Rockets employing chemical or ionic propellants require the transport of prohibitively large quantities

of propellant. If the properties of the quantum vacuum could somehow be utilized in the production of thrust, that would provide a decided advantage since the vacuum is everywhere. The three milestones for deep space or interstellar travel in the NASA Breakthrough Propulsion Physics (BPP) Project are: 1) the development of propellantless propulsion; 2) the development of methods that reduce travel time by orders of magnitude; and 3) new methods of providing the energy to operate the spacecraft[1]. At this embryonic stage in the exceedingly brief history of interstellar spacecraft, we are attempting in this paper to distinguish between what is possible and what is not possible within the context of our current understanding of the relevant physics. Science fiction writers have written about the use of the quantum vacuum for spacecraft for decades but no research has validated this suggestion. Arthur C. Clark, who proposed geosynchronous communications satellites in 1945, described a "quantum ramjet drive" in 1985 in "Songs of Distant Earth", and observed in the Acknowledgement, "If vacuum fluctuations can be harnessed for propulsion by anyone besides science-fiction writers, the purely engineering problems of interstellar flight would be solved[2]."

Since we do not know how to harness the energy of vacuum fluctuations, we have focused on a slightly different approach in this paper, and have considered how we might transfer momentum to the vacuum from the spacecraft. The spacecraft proposed in this paper, which addresses the first and last BPP milestones, is described as a "gedanken spacecraft" since its design is not intended as an engineering guide, but just to illustrate possibilities. Indeed based on our current understanding of quantum vacuum physics, one could reasonably argue that a better gedanken spacecraft could be propelled by oscillating a charged mirror or simply using a flashlight or laser to generate photons. Although the performance of the gedanken spacecraft as presented is disappointing and is no more practical than a spacewarp [3], it illustrates many current notions about the quantum vacuum, and it is interesting to understand the potential role of quantum vacuum phenomena in a macroscopic system like space travel. Obviously, we need some major breakthroughs in our understanding if we are to realize the dream of space travel as presented in science fiction works.

In 1948, when physicists were developing new ideas in quantum field theory, Lamb made a seminal measurement showing a shift in the energy levels of the hydrogen atom that was interpreted as due to the interaction of the atom with the quantum fluctuations of the vacuum state of the quantized electromagnetic field[4]. Theory showed this field produced a complex radiative shift that shifted both the energy level and its lifetime[5]. Independently, Casimir made a prediction of an attractive quantum vacuum force that would exist between two parallel, uncharged, metal plates due to the boundary conditions the quantized fluctuating vacuum electromagnetic field must meet on the surface of the plates [6]. About twelve years later, Lifshitz provided another explanation of the Casimir effect based on fluctuations in the very long range behavior of the intermolecular potentials[7]. Five decades passed before there was accurate experimental verification in the landmark experiments of Lamoroux [8] using a torsion pendulum, and Mohideen [9] using an Atomic Force Microscope (AFM). In the AFM measurements a 200 μm sphere, metallized with gold, is attached to the end of a cantilever, and moved near a flat surface. The deflection of the cantilever is measured optically, allowing the detection of forces of the order of a nN (10^{-9} N). The AFM measurements have become increasingly precise and corrections due to conductivity, temperature, and surface roughness have been developed, giving agreement between theory and experiment

at about the 1 % level[10]. Current efforts are designed to explore the temperature dependence of the Casimir force [11]. Proposals regarding the appearance of and the use of Casimir forces in micromechanical devices are being realized in microelectromechanical systems [12][13]. Buks et al have measured adhesion energies due to Casimir forces using gold films [14]. Casimir forces have recently been used to actuate a microelectromechanical torsion device [15]. The inverse fourth power behavior of the parallel plate Casimir force has been exploited to make this device operate as an anharmonic oscillator which serves as a sensitive position sensor [16]. The concept of a vacuum force that arises from the application of boundary condition has been generalized to various geometries and topologies, and has been reviewed [17][18][19].

The experimental verification of Casimir's prediction is often cited as proof of the reality of the vacuum energy density of quantum field theory. Yet, as Casimir himself observed, other interpretations are possible:

The action of this force [between parallel plates] has been shown by clever experiments and I think we can claim the existence of the electromagnetic zero-point energy without a doubt. But one can also take a more modest point of view. Inside a metal there are forces of cohesion and if you take two metal plates and press them together these forces of cohesion begin to act. On the other hand you can start with one piece and split it. Then you have first to break chemical bonds and next to overcome van der Waals forces of classical type and if you separate the two pieces even further there remains a curious little tail. The Casimir force, *sit venia verbo*, is the last but also the most elegant trace of cohesion energy [20].

The nearly infinite zero-point field energy density is a simple and inexorable consequence of quantum theory, but it brings puzzling inconsistencies with another well verified theory, general relativity [21][22]. The quantum vacuum energy density would be expected to provide a Cosmological constant of the type proposed by Einstein in order to have static solutions to his field equations. The nearly infinite vacuum energy density (10^{95} g/cc in terms of mass) predicted by quantum theory would result in an outward pressure that would rip the universe apart, yet astronomical data suggest a small astronomical constant (about 10^{-29} g/cc in terms of mass), resulting in a disagreement between two well verified theories of over 120 orders of magnitude!

Casimir effects have also been derived and interpreted in terms of *source* fields arising from fluctuations within matter in both conventional [17] and unconventional [23] quantum electrodynamics. These interpretations have spawned the evolution of two views which have proved to be equivalent in the limited systems considered to date. In one view the energy in the quantum vacuum is seen as a consequence of the fluctuations in the electrical potential from nearby matter; in the other view the energy in space is seen as due to the ground state of the quantized electromagnetic field. No experiment yet proposed has been shown to clearly distinguish between these two viewpoints, but to date only experiments on flat or nearly flat surfaces have been done. Experiments with resonant cavities may resolve some of these questions [24]. Theoretical calculations are often done using the simplest approach or the one of personal preference. In this paper, we will perform computations using the quantized vacuum field model at zero Kelvin and

refer to the properties of the quantum vacuum, with the current understanding that an equivalent derivation could probably be based on effects which arise from the tails of van der Waals forces between molecules. Intuitively one might conjecture that space travel based on the properties of the quantum vacuum is more likely to become a reality if there is actually fluctuating energy within all space, and we are not just dealing with the tails of molecular potentials of matter nearby.

Casimir effects result from changes in the ground-state fluctuations of a quantized field that occur because of the particular boundary conditions. Casimir effects occur for all quantum fields and can also arise from the choice of topology. In the special case of the vacuum electromagnetic field with dielectric or conductive boundaries, various approaches suggest that Casimir forces can be regarded as macroscopic manifestations of many-body retarded van der Waals forces [17][25].

In 1970, Moore considered the effect of an uncharged one dimensional boundary surface that moved, with the very interesting prediction that it should be possible to generate real photons from this motion [26]. Energy conservation requires the existence of a radiation reaction force working against the motion of the mirror [27]. The energy expended moving the mirror against the radiative force goes into electromagnetic radiation. This effect, generally referred to as the dynamic or adiabatic Casimir effect, has been reviewed [18][19][28]. The vacuum field exerts a force on the moving mirror that tends to damp the motion. This dissipative force may be understood as the mechanical effect of the emission of radiation induced by the motion of the mirror. The Hamiltonian is quadratic in the field operators, and formally analogous to the Hamiltonian describing photon pair creation by parametric interaction of a classical pump wave of frequency ω_o with a nonlinear medium[29]. Pairs of photons with frequencies $\omega_1 + \omega_2 = \omega_o$ are created out of the vacuum state. Furthermore the photons have the same polarization, and the components of the corresponding wave vectors \vec{k}_1 and \vec{k}_2 taken along the mirror surface must add to zero because of the translational symmetry:

$$\vec{k}_1 \cdot \hat{x} + \vec{k}_2 \cdot \hat{x} = \omega_1 \sin \theta_1 + \omega_2 \sin \theta_2 = 0. \quad (1)$$

This last equation relates the angles of emission of the photon pairs with respect to the unit vector \hat{x} , which is normal to the surface. It is interesting that the photons emitted by the dynamic Casimir effect are entangled photons. This analysis in terms of the analogous effective Hamiltonian is illuminating but not complete for perfect mirrors, because no consistent effective Hamiltonian can be constructed in this case with the idealized and pathological boundary conditions. More realistic results are obtained assuming that the mirrors are transparent above a plasma frequency.

The dynamic Casimir effect was studied for a single, perfectly reflecting mirror with arbitrary non-relativistic motion and a scalar field in three dimensions by Ford and Vilenkin[30]. They obtained expressions for the vacuum radiation pressure on the mirror. Barton and Eberlein extended the analysis using a 1 dimensional scalar field to a moving body with a finite refractive index[31]. The vacuum radiation pressure and the radiated spectrum for a non-relativistic, perfectly reflecting, infinite, plane mirror was computed by Neto and Machado for the electromagnetic field in three dimensions, and shown to obey the fluctuation-dissipation theorem from linear response theory[32][27]. This theorem shows the fluctuations for a stationary body yield information about the mean force experienced by the body in nonuniform motion. Jaekel computed shifts in the mass of the mirror for a scalar field

in two dimension[33]. The mirror mass is not constant, but rightfully a quantum variable because of the coupling of the mirror to the fields by the radiation pressure. A detailed analysis was done by Barton and Calogeracos for a dispersive mirror in 1 dimension, that includes radiative shift in the mass of the mirror and the radiative reaction force[34]. This model can be generalized to an infinitesimally thin mirror with finite surface conductivity and a normally incident electromagnetic field.

It might be of some interest to indicate why we analyze this spacecraft in terms of the dynamic Casimir effect and not the Unruh Davies radiation. Historically, Unruh and Davies made the seminal investigations into the thermal effects of acceleration on vacuum fluctuations[35][36]. They derived that a detector with constant acceleration a for *all* time (-infinity to +infinity) would observe black body radiation at temperature $\hbar a/2\pi ck$, where k is Boltzmann's constant. However, the renormalized vacuum expectation value of the stress energy momentum tensor vanishes for the field with the zero velocity detector, and by the transformation properties of tensors, it must also vanish for the accelerated state. Hence the thermal quanta that excite the accelerated detector have been described as 'fictitious' or 'quasi' thermal quanta, although this conundrum may be more a reflection of the limited meaning of the term 'particle' in accelerated reference frames or curved spaces[28]. It has been suggested that the thermal spectrum appears to arise not from the "creation of particles" but the distortion of the zero-point field[37]. Barton has observed that unresolved difficulties still obstruct any clear understanding of accounts of radiative processes referred to uniformly accelerated frames, and cites the unphysical nature of uniform acceleration for all time[31]. Hence we have adopted the language and the methods of analysis that characterize the dynamic Casimir effect in our analysis of the gedanken spacecraft.

All calculations of electromagnetic fields and radiative forces to date on moving plates have been for infinite plates. The contribution to the radiative force on a vibrating infinite plate due to electromagnetic radiation is the same for both sides of the plate[32]. From the perspective of making a numerical estimate for a finite plate, this means the theoretical results need to be adapted. To do this we assume that the plate is large enough with respect to the wavelengths of the Fourier components of the motion so that diffraction effects are small, and that the force/area for the infinite plate is approximately the result for a finite plate. Eberlein has made the very interesting suggestion that one could calculate the force for a finite two sided plate as the zero eccentricity limit of the force on an oblate spheroid[38][39].

Recently suggestions have been made to measure the motion induced radiation escaping from a cavity in which one partially transmitting wall is vibrating[40]. The rate of photon emission is enhanced by the finesse of the cavity and therefore may be orders of magnitude greater than for a plate. Plunien and his collaborators showed that the rate of photon emission by a vibrating cavity can be further enhanced by several orders of magnitude if the temperature is several hundred degrees above zero Kelvin[41]. The enhancement is proportional to the ground state photon population at the given temperature.

In Section 2, we present a simplified analysis for the use of the dynamic Casimir effect to accelerate a spacecraft. Also discussed is the possibility of powering the accelerating mirror with energy extracted from the quantum vacuum using the parallel plate Casimir force. In Section 3 a detailed description is given of a mirror trajectory that produces a net impulse during each cycle of operation. After a discussion of some possible methods to increase the thrust and impulse, the

conclusion follows.

2. A SIMPLE MODEL FOR A QUANTUM VACUUM SPACECRAFT

The important physical features of using the dynamic Casimir effect to accelerate a spacecraft can be seen in a simplified, heuristic model. Assume that the spacecraft has an energy source, such as a battery, that powers a motor that vibrates a mirror or a system of mirrors in a suitable manner to generate radiation. We will assume that there are no internal losses in the motor or energy source. We assume that at the initial time t_i , the mirrors are at rest. Then the mirrors are accelerated by the motor in a suitable manner to generate a net radiative reaction on the mirror, and at the final time t_f , the mirrors are no longer vibrating, and the spacecraft has attained a non-zero momentum. We can apply the first law of thermodynamics to the system of the energy source, motor, and mirror at times t_i and t_f :

$$\Delta Q = \Delta U + \Delta W, \quad (2)$$

where ΔU represents the change in the internal energy in the energy source, $-\Delta W$ represents the work done on the mirrors moving against the vacuum, and ΔQ represents any heat transferred between the system and the environment. We will assume that we have a thermally isolated system and $\Delta Q = 0$ so

$$0 = \Delta U + \Delta W. \quad (3)$$

By the conservation of energy, the energy ΔU extracted from the battery goes into work done on the moving mirror $-\Delta W$. Since the mirror has zero vibrational kinetic energy and zero potential energy at the beginning and the end of the acceleration period, and is assumed to operate with no mechanical friction, all work done on the mirror goes into the energy of the emitted radiation ΔR and the kinetic energy of the spacecraft of mass M :

$$\Delta W = \Delta R + M (\Delta V)^2 / 2.$$

Thus the energy of the radiation emitted due to the dynamic Casimir effect equals

$$\Delta R = -\Delta U - M (\Delta V)^2 / 2 > 0. \quad (4)$$

The frequency of the emitted photons depends on the Fourier components of the motion of the mirror. We assume that the radiant energy can be expressed as a sum of energies of n_i photons each with frequency ω_i :

$$\Delta R = \sum_i n_i \hbar \omega_i. \quad (5)$$

Let us assume that all photons are emitted normally from one side of the accelerating surface. This assumption is not valid, as Eq.1 and the work of Neto and Machado [27] clearly show, but it allows us to obtain a best case scenario and illustrates the main physical features. If all photons are emitted normally from one

surface, then the photon momentum transfer ΔP is :

$$\Delta P = \sum_i n_i \frac{\hbar \omega_i}{c} = \frac{\Delta R}{c}, \quad (6)$$

where c is the speed of light. Using Eq. 4, we obtain the result:

$$\Delta P = \frac{-\Delta U}{c} - \frac{M(\Delta V)^2}{2c}. \quad (7)$$

In a non-relativistic approximation $\Delta P = M\Delta V$ and the change in velocity ΔV of the spacecraft is to second order in $\Delta U/Mc^2$:

$$\frac{\Delta V}{c} = \frac{-\Delta U}{Mc^2} + \left(\frac{\Delta U}{Mc^2} \right)^2. \quad (8)$$

This represents a maximum change in velocity attainable by use of the dynamic Casimir effect (or by the emission of electromagnetic radiation generated by more conventional means) when the energy ΔU is expended. The ratio $\Delta U/Mc^2$ is expected to be a small number, and we can neglect the second term in Eq. 8. (As a point of reference, for a chemical fuel the ratio of the heat of formation to the mass energy is approximately 10^{-10} .) With this approximation, we find the maximum value of $\Delta V/c$ equals $\Delta U/Mc^2$, the energy obtained from the energy source divided by the rest mass energy of the spacecraft. It follows that the kinetic energy of the motion of the spacecraft E_{ke} can be expressed as

$$E_{ke} = \frac{M(\Delta V)^2}{2} = \Delta U \frac{\Delta U}{2Mc^2}. \quad (9)$$

This result shows that the conversion of potential energy ΔU from the battery into kinetic energy of the spacecraft is an inefficient process since $\Delta U/Mc^2$ is a small factor. Almost all of the energy ΔU has gone into photon energy. This inefficiency follows since the ratio of momentum to energy for the photon is $1/c$.

In our derivation, no massive particles are ejected from the space craft (propellantless propulsion) and we have neglected: 1. the change in the mass of the spacecraft as the stored energy is converted into radiation, 2. radiative mass shifts, 3. complexities related to high energy vacuum fluctuations and divergences, 4. all dissipative forces in the system used to make the mirror vibrate. These assumptions are consistent with a heuristic non-relativistic approximation.

In this simplified model, we have not made any estimates about the rate of photon emission and how long it would take to reach the maximum velocity. Rates are typically very low, typically 10^{-5} photons/sec [27].

2.1. Use of the static Casimir Effect as an energy Source

In order to explore the possibility of a spacecraft that is based completely on quantum vacuum properties, consider the use of an arrangement of perfectly conducting, uncharged, parallel plates in vacuum as an energy source. The Casimir energy $U_C(x)$ at zero degrees Kelvin between plates of area A , separated by a distance x is:

$$U_C(x) = -\frac{\pi^2}{720} \frac{\hbar c A}{x^3}. \quad (10)$$

If we allow the plates to move from a large initial separation a to a very small final separation b then the change in the vacuum energy between the plates is approximately:

$$\Delta U_C = U_C(b) - U_C(a) \quad (11)$$

$$\approx -\frac{\pi^2}{720} \frac{\hbar c A}{b^3}. \quad (12)$$

The attractive Casimir force has done work on the plates, and, in principle, we can build a device to extract this energy with a suitable, reversible, isothermal process, and use it to accelerate the mirrors. We neglect any dissipative forces in this device, and assume all of the energy ΔU_C can be utilized. Thus the maximum value of $\frac{\Delta V_C}{c}$ obtainable using the energy from the Casimir force "battery" is

$$\frac{\Delta V_C}{c} = \frac{\pi^2}{720} \frac{1}{Mc^2} \frac{\hbar c A}{b^3}. \quad (13)$$

We can make an upper bound for this velocity by making further assumptions about the composition of the plates. Assume that the plate of thickness L is made of a material with a rectangular lattice that has a mean spacing of d , and that the mass associated with each lattice site is m . Then the mass of one plate is:

$$M_P = AL \frac{m}{d^3}. \quad (14)$$

The density approximation is good for materials with a cubic lattice, and within an order of magnitude of the correct density for other materials.

In principle, it is possible to make one of the plates in the battery the same as the plate accelerated to produce radiation by the dynamic Casimir effect. As the average distance between the plates is decreased, the extracted energy is used to accelerate the plates over very small amplitudes. If we assume we need to employ two plates in our spacecraft, and that the assembly to vibrate the plates has negligible mass, then the total mass of the spacecraft is $M = 2M_P$ and we obtain an upper limit on the increase in velocity:

$$\frac{\Delta V_C}{c} = \frac{\pi^2}{1440} \frac{\hbar}{Lmc} \frac{d^3}{b^3}. \quad (15)$$

The final velocity is proportional to the Compton wavelength (\hbar/mc) of the lattice mass m divided by the plate thickness L . Assume that the final spacing between the plates is one lattice constant ($d = b$), that the lattice mass m equals the mass of a proton m_p , and that the plate thickness L is one Bohr radius $a_o = \hbar^2/m_e e^2$, then we obtain (α is the fine structure constant with approximate value of $1/137$):

$$\frac{\Delta V_C}{c} = \frac{\pi^2}{1440} \frac{\alpha m_e}{m_p}. \quad (16)$$

(A real plate made using current technology might easily be three orders of magnitude thicker.) Substituting numerical values we find:

$$\frac{\Delta V_C}{c} = \frac{\pi^2}{1440} \frac{1}{137} \frac{1}{1800} = 2.78 \times 10^{-8}. \quad (17)$$

This best case scenario corresponds to a disappointing final velocity of about 8 m/s, about 10^3 times smaller than for a large chemical rocket. As anticipated, the spacecraft is very slow despite the unrealistically favorable assumptions made in the calculation, yet this simple gedanken experiment does demonstrate that it may be possible to base the operation of a spacecraft entirely on the properties of the quantum vacuum.

2.2. Comments on the possible Use of vacuum Fluctuations as an energy Source

Let us briefly comment on the long range possibility of extracting larger amounts of energy from vacuum fluctuations. If, for example, it were possible to make a structure equivalent to a parallel plate structure that could have a final state with a separation 100 or 1000 times smaller than as assumed above, then we would have the suggestion of a spacecraft operating in the relativistic regime. Without a new approach, such a development is probably not possible since the Casimir force decreases for separations less than the plasma wavelength. (Several new approaches are discussed in Section 3.1.) The reflectivity of a metal plate depends on the applied frequency, and for frequencies above the plasma frequency the reflectivity drops, causing the Casimir force to decrease. Alternatively, if there were a device that could continuously extract energy from the quantum fluctuations of the vacuum field, then this vacuum powered spacecraft, and a lot of other unusual things, might become practical.

It is important to note that utilizing energy of the quantum fluctuations of the electromagnetic field does not appear to directly violate known laws of physics, based on the work of Forward and Cole and Puthoff, however improbable or impossible such a development might seem [42][43]. Forward showed that it is possible to conceive of a device, a foliated capacitor, in which one could extract electrical energy from the quantum vacuum to do work. The energy is extracted as the portions of the capacitor that repel each other due to electrostatic forces come together under the influence of the Casimir force[42]. Cole and Puthoff used stochastic electrodynamics to examine the process of removing energy from the vacuum fluctuations at zero temperature from the viewpoint of thermodynamics and showed there is no violation of the laws of thermodynamics[43]. In the same spirit, Rueda has suggested that very high energy particles observed in space may derive their kinetic energy from a long term acceleration due to the stochastic vacuum field[44]. In a careful analysis, Cole has shown that this process of energy transfer from the vacuum field to kinetic energy of the particles does not violate the laws of thermodynamics[45]. In stochastic electrodynamics one treats the vacuum fluctuations as a universal random classical electromagnetic field. A formal analogy exists between stochastic electrodynamics and quantum electrodynamics: the field correlation functions in one theory are related to the Wightman functions in the other theory[46]. Pinto has done a calculation of a solid state Casimir device which is described as a transducer of vacuum energy which can operate in a repetitive cycle [47].

It is our opinion that further research is needed to gain a greater understanding of what is possible and what are the fundamental limitations and restrictions in processes involving energy transfer with the quantum vacuum. One can take an alternative viewpoint here, that we are just extracting energy from the tails of the molecular potentials, and that much more energy would be available if we utilized the entire potential, for example, by burning the material. However, this approach

just leads us back to the known limitations of chemical fuels.

3. DETAILED MODEL FOR PROPULSION USING DYNAMIC CASIMIR EFFECT

Assume we have a flat, perfectly reflecting, mirror whose equilibrium position is $x = 0$. At a time t where $t_i < t < t_f$ the location of the mirror is given by $x(t)$. Neto has given an expression for the force per unit area $F(t)$ on such a mirror [32]:

$$F(t) = \lim_{\delta x \rightarrow 0} \frac{\hbar c}{30\pi^2} \left[\frac{1}{\delta x} \frac{d^4 x(t)}{c^4 dt^4} - \frac{d^5 x(t)}{c^5 dt^5} \right], \quad (18)$$

where δx represents the distance above the mirror at which the stress-energy tensor is evaluated. The second term represents the dissipative force that is related to the creation of travelling wave photons, in agreement with its interpretation as a radiative reaction. In computing the force due to the radiation from the mirror's motion, the effect of the radiative reaction on $x(t)$ is neglected in the nonrelativistic approximation. The divergent first term can be understood in several ways. Physically it is a dispersive force that arises from the scattering of low frequency evanescent waves. The divergence can be related to the unphysical nature of the perfect conductor boundary conditions. Forcing the field to vanish on the surface requires its conjugate momentum to be unbounded. Thus the average of the stress-energy tensor $\langle T_{\mu\nu} \rangle$ is singular at the surface for the same reason that single-particle quantum mechanics would require a position eigenstate to have infinite energy [30]. This divergent term can be lumped into a mass renormalization, and therefore disappears from the dynamical equations when they are expressed in terms of the observed mass of the body [31][33]. We will not discuss this term further in this calculation, although we will return to the general idea of radiative mass shift in our discussion. We will assume that diffraction effects are small for our finite plates.

The total energy radiated per unit plate area E can be expressed as

$$E = - \int_{t_1}^{t_2} dt F(t) \frac{dx(t)}{dt}. \quad (19)$$

Substituting Eq. 18 for $F(t)$ we find

$$E = \frac{\hbar}{30\pi^2 c^4} \int_{t_1}^{t_2} dt \left(\frac{d^3 x(t)}{dt^3} \right)^2. \quad (20)$$

The total impulse I per unit plate area can also be computed as the integral of the force per unit area over time:

$$I = \int_{t_1}^{t_2} dt F(t) = - \frac{\hbar}{30\pi^2 c^4} \left(\frac{d^4 x(t)}{dt^4} \Big|_{t_2} - \frac{d^4 x(t)}{dt^4} \Big|_{t_1} \right). \quad (21)$$

The total impulse I equals the mass of the system M per unit area times the change

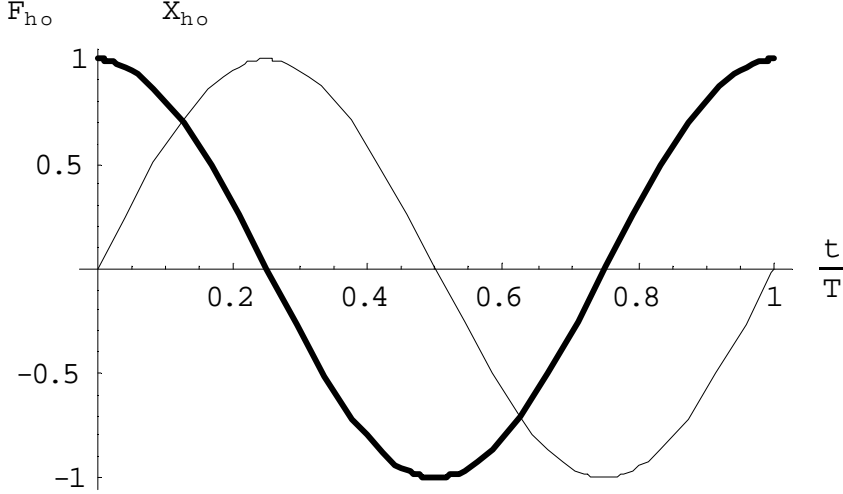


FIG. 1 The displacement x_{ho} and the radiative reaction force F_{ho} , bold line, for a harmonically oscillating mirror plotted as a function of the time. For convenience F_{ho} and x_{ho} are normalized to unity amplitude.

in velocity ΔV in a non-relativistic approximation:

$$I = M\Delta V. \quad (22)$$

We want to specify a trajectory for the mirror that will give a net impulse. One of the trajectories that has been analyzed is that of the harmonic oscillator[40][30]. In this case, the mirror motion is in a cycle and we can compute the energy radiated per cycle per unit area and the impulse per cycle per unit area. For a harmonic oscillator of frequency Ω and period $T = 2\pi/\Omega$, there is only one Fourier component of the motion, so the total energy of each pair of photons emitted is $\hbar\Omega = \hbar(\omega_1 + \omega_2)$. For a harmonically oscillating mirror the displacement is

$$x_{ho}(t) = X_o \sin \Omega t. \quad (23)$$

A computation based on Eqs. 20 and 21 shows there will be a net power radiated in a cycle, however, the dissipative force for the harmonic oscillator F_{ho} will average to zero over the entire cycle as shown in Fig. 1, so there will be no net impulse.

In order to secure a net impulse, we need a modified mirror cycle. One such model cycle can be readily constructed by using the harmonic function $x_{ho}(t)$ over the first and last quadrants of the cycle, where the force F_{ho} is positive, and a cubic function $x_c(t)$ over the middle two quadrants where F_{ho} is negative:

$$x_c(t) = \frac{X_o}{2} \frac{(\Omega t - \pi)^3}{(\pi/2)^3} - \frac{3X_o}{2} \frac{(\Omega t - \pi)}{\pi/2}. \quad (24)$$

The coefficients for the cubic polynomial are chosen so that at $\Omega t = \pi/2, 3\pi/2$ the displacement and the first derivatives of $x_c(t)$ and $x_{ho}(t)$ are equal. As can be seen

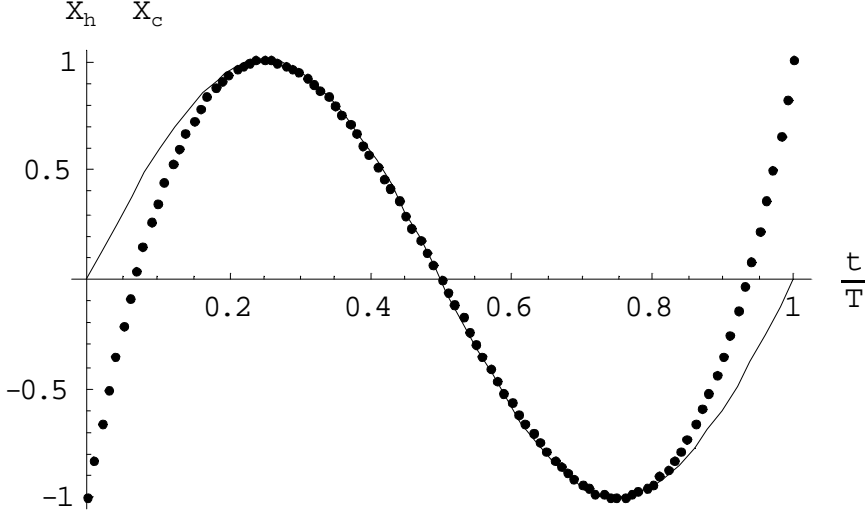


FIG. 2 The normalized displacement for a harmonically oscillating mirror $x_{ho}(t)$, solid line, and the cubic function $x_c(t)$, shown by the bold dots.

from Fig. 2, the cubic function $x_c(t)$ matches $x_{ho}(t)$ quite closely for t in the interval $.25 < t/T < .75$. Of course the higher order derivatives do not match, and that is precisely why the force differs. The similarity in displacement and the difference in the resulting force is striking. For the mirror displacement $x_m(t)$ in our model we choose:

$$x_m(t) = x_{ho}(t) \quad \text{for } 0 \leq t/T \leq 0.25; 0.75 \leq t/T \leq 1, \quad (25)$$

$$x_m(t) = x_c(t) \quad \text{for } 0.25 < t/T < 0.75. \quad (26)$$

Fig. 3 shows $x_m(t)$ plotted with the corresponding force per unit area $F_m(t)$ obtained from Eq. 18. The force $F_m(t)$ is positive in the first and last quarter of the cycle, and vanishes in the middle, where the trajectory is described by the cubic. The energy radiated per area per cycle for our model trajectory can be obtained from Eq. 20:

$$E_m = -\frac{\hbar c}{60\pi} X_o^2 \left(\frac{\Omega}{c}\right)^5. \quad (27)$$

The total impulse per area per cycle for our model I_m trajectory is

$$I_m = -\frac{\hbar}{15\pi^2} X_o \left(\frac{\Omega}{c}\right)^4. \quad (28)$$

The impulse is first order in \hbar , and is therefore typically a small quantum effect. Thus for our model cycle, the change in velocity per second is $\Delta V_m/dt$:

$$\Delta V_m/dt = \frac{I_m \Omega}{M}, \quad (29)$$

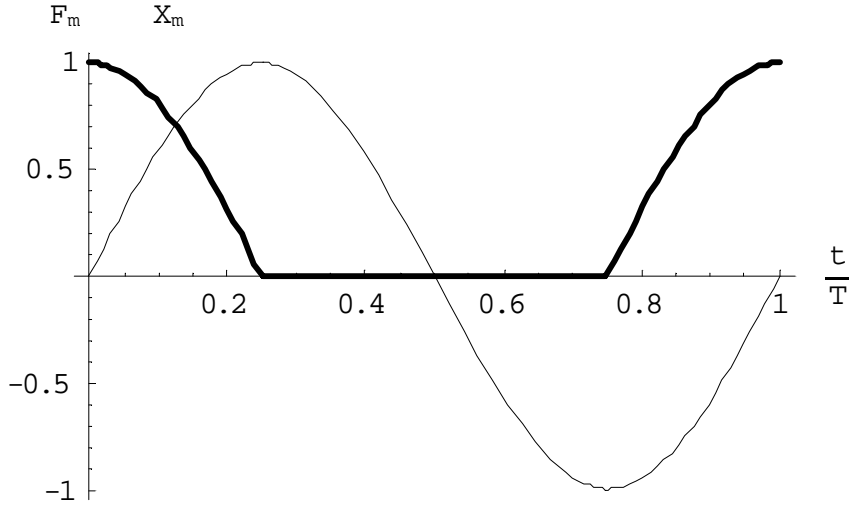


FIG. 3 The normalized displacement $x_m(t)$ and the corresponding normalized radiative force $F_m(t)$, bold solid line, are shown as functions of the time. The force is positive in the first and last quarters, and zero in the middle half of the cycle.

where M is the mass per unit plate area of the spacecraft, and we assume the plate is the only significant mass in the gedanken spacecraft. In order to estimate ΔV_m , we can make some further assumptions regarding the mass of the plate per unit area. As before, we can make a very favorable assumption regarding the mass per unit area of the plates $M = m_p/a_o^2$, which yields the change in velocity per second:

$$\Delta V_m/dt = -\frac{\hbar}{15\pi^2} X_o \left(\frac{\Omega}{c}\right)^4 \Omega \frac{a_o^2}{m_p}. \quad (30)$$

If we substitute reasonable numerical values[24][40], a frequency of $\Omega = 3 \times 10^{10} \text{ s}^{-1}$ and an oscillation amplitude of $X_o = 10^{-9} \text{ m}$, we find that $\Delta V_m/dt$ is approximately $3 \times 10^{-20} \text{ m/s}^2$ per unit area, not a very impressive acceleration. Physically, one would imagine the surface of the mirror vibrating with an amplitude of one nanometer. The limitation in the amplitude arises because the maximum velocity of the boundary is proportional to the elastic deformation, which cannot exceed about 10^{-2} for typical materials. The energy radiated per area $E_m \Omega$ is about 10^{-25} W/m^2 . There are a number of methods to increase these value by many orders of magnitude, as discussed below.

The efficiency of the conversion of energy expended per cycle in our model E_m into kinetic energy of the spacecraft $E_{ke} = \frac{1}{2} M (\Delta V_m)^2 = I_m^2 / 2M$ is given in the nonrelativistic approximation by the ratio:

$$\frac{E_{ke}}{E_m} = \frac{\hbar}{Mc\pi} \left(\frac{\Omega}{c}\right)^3. \quad (31)$$

With our assumptions, the approximate value of this ratio is 10^{-26} , making this

conversion an incredibly inefficient process.

3.1. Methods to Increase Acceleration

The dynamic Casimir effect has yet to be verified experimentally. Hence there have been a number of interesting proposals describing methods designed to maximize the effect so it can be measured. In 1994, Law predicted a resonant response of the vacuum to an oscillating mirror in a one-dimensional cavity [48]. The behavior of cavities formed from two parallel mirrors that can move relative to each other is qualitatively different from that of single plates. For example it is possible to create particles in a cavity with plates separating at constant velocity [49]. The very interesting proposal by Lambrecht et al concludes that if the mechanical oscillation frequency is equal to an odd integer multiple of the fundamental optical resonance frequency, then the rate of photon emission from a vibrating cavity formed with walls that are partially transmitting with reflectivity r_1 and r_2 , is enhanced by a factor equal to the finesse $f = 1/\ln(1/r)$ of the cavity [40][24]. For semiconducting cavities with frequencies in the GHz range, the finesse, which can be 10^9 , giving our gedanken spacecraft an acceleration of $3 \times 10^{-11} \text{ m/s}^2$ based on Eq. 30. Plunien et al have shown that the resonant photon emission from a vibrating cavity is further increased if the temperature is raised [41]. For a 1 cm cavity, the enhancement is about 10^3 for a temperature of $290 \text{ }^\circ\text{K}$. Assume that one has a gedanken spacecraft with a vibrating cavity operating at about $290 \text{ }^\circ\text{K}$ providing a 10^{12} total increase in the emission rate. This would result in an acceleration per unit area of the plates of $3 \times 10^{-8} \text{ m/s}^2$, a radiated power of about 10^{-13} W/m^2 , and an efficiency E_{ke}/E_m of about 10^{-14} . After 10 years of operation, the gedanken spacecraft velocity would be approximately 10 m/s , which is about 3 orders of magnitude less than the current speed of Voyager, 17 km/s , obtained after a gravity assist maneuver around Jupiter to increase the velocity. (The burn-out velocity for Voyager at launch in 1977 was 7.1 km/s [50].)

The numerical results for the model obviously depend very strongly on the assumptions made. For the plate mass/area and system to vibrate the plate, we have made unrealistically favorable estimates; for the oscillation frequency and amplitude we have made realistic estimates. It is possible that new materials, with the ability to sustain larger strains, could make possible an amplitude of oscillation orders of magnitude larger than one nanometer. Perhaps the use of nanomaterials, such as carbon nanotubes that support 5 % strain [51], or "super" alloys [52] would allow a much larger effective deformation. If the amplitude was 1 mm instead of 1 nm, the gedanken spacecraft, or some modification with improved coupling to the vacuum, might warrant practical consideration.

Eberlein has shown that density fluctuations in a dielectric medium would also result in the emission of photons by the dynamic Casimir effect [53]. This approach may ultimately be more practical with large area dielectric surfaces driven electrically at high frequencies. More theoretical development is needed to evaluate the utility of this method. Other solid state approaches may also be of value with further technological developments. For example, one can envision making sheets of charge that are accelerated in MOS type structures. Yablonovitch has pointed out that the zero-point electromagnetic field transmitted through a window whose index of refraction is falling with time shows the same phase shift as if it were reflected from an accelerating mirror [54]. He suggested utilizing the sudden change in refractive index that occurs when a gas is photoionized or the sudden creation

of electron-hole pairs in a semiconductor, which can reduce the index of refraction from ~ 3.5 to 0 in a very short time. Using subpicosecond optical pulses, the phase modulation can suddenly sweep up low-frequency waves by many octaves. By lateral synchronization, the moving plasma front can in effect act as a moving mirror exceeding the speed of light. Therefore one can regard such a gas or semiconductor slab as an observational window on accelerating fields, with accelerations as high as $\sim 10^{20}$ m/sec [54]. Accelerations of this magnitude will have very high frequency Fourier components. Eq. 30 shows that the impulse goes as the fourth power and the efficiency as the third power of the Fourier component for an harmonic oscillator which suggests that with superhigh accelerations, an optimum time dependence of the field, and an optimum shape of the wavefronts, one might be able to secure much higher fluxes of photons and a much higher impulse/second, with a higher conversion efficiency. On the other hand, a preliminary calculation by Lozovik et al to investigate this approach suggests that the accelerated plasma method may have limitations in producing photons.[55]

Ford and Svaiter have shown that it may be possible to focus the fluctuating vacuum electromagnetic field [56]. This capability might be utilized to create regions of higher energy density. This might be of use in a cavity in order to increase the flux of radiated photons. There may be also enhancements due to nature of the index of refraction for real materials. For example, Ford has computed the force between a dielectric sphere, whose dielectric function is described by the Drude model, and a perfectly reflecting wall, with the conclusion that certain large components of the Casimir force no longer cancel. He predicts a dominant oscillatory contribution to the force, in effect developing a model for the amplification of vacuum fluctuations. Barton and Eberlein have shown that for materials with a fixed index of refraction, the force for a one dimensional scalar field goes as $[(n-1)/n]^2$, which suggest the possibility that one might be able to enhance the force by selection of a material with a small index [31].

Another, albeit improbable, approach to a vacuum facilitated gedanken spacecraft is to consider the possibility of adjusting the radiative mass shift, which we have neglected until this point. There is a very small radiative shift of the mirror due to its interaction with the vacuum, akin to the Lamb shift for an atom [40][34]. To measure a vacuum mass shift, a proposal was made recently to measure the inertial mass shift in a multilayer Casimir cavity, which consists of 10^6 layers of metal 100 nm thick, 35 cm in diameter, alternating with films of silicon dioxide 5 nm thick [61]. The mass shift is anticipated to arise from the decrease in the vacuum energy between the parallel plates. A calculation shows that the mass shift for the proposed cavity is at or just beyond the current limit of detectability. It appears that if quantum vacuum engineering of spacecraft is to become practical, and the dreams of science fiction writers are to be realized, we may need to develop new methods to be able to manipulate changes in vacuum energy densities that are near to the same order of magnitude as mass energy densities. Then we would anticipate being able to shift inertial masses by a significant amount. Since mass shifts in computations are often formally infinite, perhaps such developments are not forbidden. With large mass shifts one might be able to build a structure that had a small or zero inertial mass, which could be readily accelerated.

4. CONCLUSION

The primary objective in this paper is to illustrate some of the unique properties of the quantum vacuum and how they might be utilized in a gedanken spacecraft. We have demonstrated that it is possible in principle to cause a spacecraft to accelerate due to the dissipative force an accelerated mirror experiences when photons are generated from the quantum vacuum. Further we have shown that one could in principle utilize energy from the vacuum fluctuations to operate the vibrating mirror assembly required. The application of the dynamic Casimir effect and the static Casimir effect may be regarded as a proof of principle, with the hope that the proven feasibility will stimulate more practical approaches exploiting known or as yet unknown features of the quantum vacuum. A model gedanken spacecraft with a single vibrating mirror was proposed which showed a very unimpressive acceleration due to the dynamic Casimir effect of about $3 \times 10^{-20} \text{ m/s}^2$ with a very inefficient conversion of total energy expended into spacecraft kinetic energy. Employing a set of vibrating mirrors to form a parallel plate cavity and raising the cavity temperature to about 290^0 K increases the output by a factor of the finesse of the cavity times approximately 10^3 yielding an acceleration of about $3 \times 10^{-8} \text{ m/s}^2$ and a conversion efficiency of about 10^{-14} . After 10 years at this acceleration, the spacecraft would be traveling at 10 m/s .

From the status of current research in Casimir forces, it is clear that we understand little of the properties of the quantum vacuum for real systems with real material properties. For example, there is no general agreement with regard to the calculations of static vacuum forces for geometries other than infinite parallel plates of ideal or real metals at a temperature of absolute zero. Non-zero temperature corrections for flat, real metals are uncertain [57][58]. There are fundamental disagreements about the computation of vacuum forces for spheres or rectangular cavities, about how to handle real material properties and curvature in these and other geometries [59]. Indeed, it is very difficult to calculate Casimir forces for these simple geometries and to relate the calculations to an experiment. Calculations have yet to be done for more complex, and potentially interesting geometries. The usual problems in QED, such as divergences due to unrealistic boundary conditions, to curvature, to interfaces with different dielectric coefficients, etc. abound [60].

The vacuum effects which we computed scale with Planck's constant and so are very small. In order to have a practical spacecraft based on quantum vacuum properties, it would probably have to be based on phenomena that scale as \hbar^0 . By itself this requirement does not guarantee a large enough magnitude, but it helps [40]. New methods of modifying the quantum vacuum boundary conditions may be needed to generate the large changes in the free field vacuum energy or momentum required if "vacuum engineering" as proposed in this paper is ever to be practical. For example, the vacuum energy density between parallel plates, by which we actually mean the difference between the vacuum energy density in free space and vacuum energy density between the plates, is simply not large enough in magnitude for our engineering purposes. Energy densities, positive or negative, that are orders of magnitude greater are required. Such energy density regions may be possible, at least in some cases. For example, a region appeared in the 1 dimensional dynamic system in which the energy density was below that of the Casimir parallel plate region [48]. Similarly, more effective ways of transferring momentum to the quantum vacuum than the adiabatic Casimir effect are probably

necessary if a spacecraft is to be propelled using the vacuum.

Although the results of our calculations using the adiabatic Casimir force to propel a gedanken rocket are very unimpressive, it is important to not take our conclusions regarding the final velocity in our simplified models too seriously. The choice of numerical parameters can easily affect the final result by 4 orders of magnitude. The real significance of the result is that a method has been described that illustrates the possibility of propelling a rocket by coupling to the vacuum. It is possible that there may be vastly improved methods of coupling. In about 1900 an article was published in Scientific American proving that it was impossible to send a rocket, using a conventional propellant, to the moon. The result was based on the seemingly innocuous assumption of a single stage rocket.

ACKNOWLEDGMENTS

We gratefully acknowledge helpful comments and suggestions from Dan Cole, and thank Gabriel Barton, Carlos Villarreal Lujan, Claudia Eberlein, Peter Milonni, and Paulo Neto for discussions about their work. I also thank the reviewers for very helpful comments and suggestions. We would like to express our appreciation to the NASA Breakthrough Propulsion Physics Program and Marc Millis for the support of this work. We are very sad to report that Robert L Forward died during the process of preparing this paper.

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