

# Of Some Theoretical Significance: Implications of Casimir Effects

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## Abstract

In his autobiography Casimir barely mentioned the Casimir effect, but remarked that it is “of some theoretical significance” [1]. We will describe some aspects of Casimir effects that appear to be of particular significance now, more than half a century after Casimir’s famous paper [2].

## 1 Introduction

Let us first recall that Casimir discovered his effect as a byproduct of some applied industrial research in the stability of colloidal suspensions used to deposit films in the manufacture of lamps and cathode ray tubes. In the 1940s J.T.G. Overbeek at the Philips Laboratory studied the properties of suspensions of quartz powder, and experiments indicated that the theory of colloidal stability he had developed with E.J.W. Verwey could not be entirely correct. Better agreement between theory and experiment could be obtained if the interparticle interaction somehow fell off more rapidly at large distances than had been supposed. Overbeek suggested that this might be related to the finite speed of light,

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and his suggestion prompted his co-workers Casimir and Polder to reconsider the theory of the van der Waals interaction with retardation included. They concluded that Overbeek was right: retardation causes the interaction to vary as  $r^{-7}$  rather than  $r^{-6}$  at large intermolecular separations  $r$ .

Intrigued by the simplicity of the result, Casimir sought a deeper understanding. A conversation with Bohr led him to an interpretation in terms of zero-point energy. Then he went further with the idea of zero-point energy and showed that two perfectly conducting parallel plates should be attracted to each other as a consequence of the change they create in zero-point field energy:

Summer or autumn 1947 (but I am not absolutely certain that it [was] not somewhat earlier or later) I mentioned my results to Niels Bohr, during a walk. That is nice, he said, that is something new. I told him that I was puzzled by the extremely simple form of the expressions for the interaction at very large distance and he mumbled something about zero-point energy. That was all, but it put me on a new track.

I found that calculating changes of zero-point energy really leads to the same results as the calculations of Polder and myself ...

On May 29, 1948 I presented my paper "On the attraction between two perfectly conducting plates" at a meeting of the Royal Netherlands Academy of Arts and Sciences. It was published in the course of the year ... [3]

At about the same time the observation of the Lamb shift led to the interpretation of that effect in terms of changes in zero-point energy, or vacuum fluctuations, but Casimir's thinking was independent of this development:

... I was not at all familiar with [that work]. I went my own, somewhat clumsy way ... I do not think there were outside influences ... [3].

The Casimir force between conducting plates is a more palpable consequence of zero-point field than, for instance, the Lamb shift. It is perhaps for this reason that it now appears to be the most widely cited example of how vacuum fields and their fluctuations can have observable manifestations. The current interest owes much to recent experiments that unambiguously confirm Casimir's prediction and allow the experimental investigation of such things as finite conductivity and temperature corrections to the Casimir force between plates [4], [5].

The experimental verification of Casimir's prediction is often cited as proof of the reality of the vacuum energy density of quantum field theory. However, Casimir effects have also been derived and interpreted in terms of *source* fields in both conventional [6] and nonconventional [7] quantum electrodynamics. Various other, perhaps less fundamental interpretations are also possible, as Casimir himself observed:

The action of this force [between parallel plates] has been shown by clever experiments and I think we can claim the existence of the

electromagnetic zero-point energy without a doubt. But one can also take a more modest point of view. Inside a metal there are forces of cohesion and if you take two metal plates and press them together these forces of cohesion begin to act. On the other hand you can start with one piece and split it. Then you have first to break chemical bonds and next to overcome van der Waals forces of classical type and if you separate the two pieces even further there remains a curious little tail. The Casimir force, *sit venia verbo*, is the last but also the most elegant trace of cohesion energy [8].

Casimir effects result from changes in the ground-state fluctuations of a quantized field that occur due to the boundary conditions. Casimir effects occur for all quantum fields and can also arise from the choice of topology. In the special case of the vacuum electromagnetic field with dielectric or conductive boundaries, various approaches suggest that Casimir forces can be regarded as macroscopic manifestations of many-body retarded van der Waals forces [6], [9].

Zero-point field energy density is a simple and inexorable consequence of quantum theory, but it brings puzzling inconsistencies with another well verified theory, general relativity. The total energy density of the vacuum would be expected to provide a cosmological constant of the type introduced by Einstein in order to have static solutions of his field equations. The predicted electromagnetic quantum vacuum energy density is enormous (about  $10^{114}$  J/m<sup>3</sup> or, in terms of mass,  $10^{95}$  g/cm<sup>3</sup> if the Planck length of  $10^{-35}$  m is used to provide a cut-off), and for an infinite flat universe would imply an outward zero-point pressure that would rip the universe apart [10]. Astronomical data, on the other hand, indicate that any such cosmological constant must be  $\sim 4$  eV/mm<sup>3</sup>, or  $10^{-29}$  g/cm<sup>3</sup> when expressed as mass [11]. The discrepancy between theory and observation is about 120 orders of magnitude, arguably the greatest quantitative discrepancy between theory and observation in the history of science [12], [13]! There are numerous approaches to solve this “cosmological constant problem,” such as renormalization, supersymmetry, string theory, and quintessence, but as yet this remains an unsolved problem.

The Casimir effect is important as well in connection with other aspects of cosmology and space-time physics. Fluctuations in vacuum field energy are believed by cosmologists to be responsible for the origin of the universe. These fluctuations may have provided the primordial irregularities required to form stars and galaxies, and may be the source of the cosmic temperature fluctuations uncovered by the COBE satellite in 1992.

The possibility of a “traversable wormhole” tunnel in space-time [14] is attributable to the modification of the vacuum by the Casimir effect, and in particular to the negative energy density between the caps of the wormhole. There is an interesting question, however, about whether the positive energy density associated with the caps will result in a net energy density that is insufficiently negative (Visser 1996, pp 121-6).

Observable consequences of focusing vacuum fluctuations with a parabolic

reflector have recently been predicted [15]. Casimir effects can also arise from dynamical constraints, such as moving mirrors or varying gravitational fields, that alter the vacuum. A sudden displacement of a reflecting boundary, for instance, is not communicated to a point at a distance  $d$  from the boundary until a time  $d/c$ , and a consequence of this is that radiation is generated, i.e., particles are created. The effect is very weak unless enormous accelerations are imagined. However, if one of the plates in the original Casimir example is made to oscillate resonantly with the photon propagation time in the cavity, there is an amplification of the effect that might make the radiation observable [16]. Many of the recent predictions of vacuum-field effects are, to say the least, not readily observable [17], [18]. The significance of Casimir's work in this context is that it makes an experimentally verifiable prediction based on the quantum vacuum, and thereby lends support to these other predictions that rely on quantum vacuum theory.

Another vacuum effect that has received much attention is the Unruh-Davies effect: a detector (or atom) moving with uniform acceleration in the vacuum responds as if it is at rest in a thermal field of temperature  $T = \hbar a / 2\pi k c$ , where  $a$  is the proper acceleration and  $k$  is Boltzmann's constant. Vacuum fluctuations are in effect promoted to thermal fluctuations. Unfortunately the accelerations required for one to seriously contemplate an experimental observation of the effect are prohibitively large. (A temperature 1 pK corresponds to an acceleration of  $2.5 \times 10^8$  m/sec<sup>2</sup>.)

## 2 Vacuum Friction

Consider instead the case of an atom moving in an isotropic *thermal* field. In this case there is an effect that depends on the atom's *velocity*: an atom with velocity  $v$  experiences a drag force

$$F = - \left( \frac{\hbar \omega}{c^2} \right) (p_1 - p_2) B_{12} \left( \rho(\omega) - \frac{\omega}{3} \frac{d\rho}{d\omega} \right) v, \quad (1)$$

where  $\omega$  is the transition frequency between the lower state 1 and the upper state 2,  $p_1, p_2$  are the state occupation probabilities,  $B_{12}$  is the Einstein  $B$  coefficient for absorption, and  $\rho(\omega)$  is the spectral energy density of the field. (For simplicity we restrict ourselves to two nondegenerate energy levels of the atom.) This result was obtained by Einstein [19], who showed that the increase in the atoms' kinetic energy upon absorption and emission of radiation is balanced on average by the drag force if the equilibrium  $\rho(\omega)$  is the Planck spectrum.

What does this have to do with Casimir effects or the vacuum? Let us note first that, for the vacuum spectral energy density  $\rho_0(\omega) = \hbar \omega^3 / 2\pi^2 c^3$ , the drag force vanishes. This is as it should be: Lorentz invariance of the vacuum does not allow a velocity-dependent force. But what happens if we arrange for the zero-temperature spectral density of the electromagnetic field to be different from the  $\rho_0(\omega)$  of infinite free space?

One way to obtain a zero-temperature spectral density different from  $\rho_0(\omega)$ , of course, is to introduce conducting surfaces. Local changes in mode density and therefore vacuum energy density are induced by the presence of curved surfaces, and, depending on whether the curvature is positive or negative, the force between the surface and the particle may be repulsive or attractive [20]. Indeed, whenever there is an inhomogeneous vacuum energy density, there will be a net force on a polarizable neutral particle given by  $\frac{1}{2}\alpha\vec{\nabla}\langle E(x)^2\rangle$ . The simplest example of using a surface to alter vacuum modes is a perfectly conducting, infinite wall. The change in the vacuum field energy due to the wall produces in this case the well-known Casimir-Polder interaction: for sufficiently large distances  $d$  from the wall this interaction is  $V(d) = -3\alpha\hbar c/8\pi d^4$ , where  $\alpha$  is the static polarizability of the (ground-state) atom. This effect has been accurately verified in the elegant experiments of Sukenik *et al* [21].

Now let the atom move parallel to the wall with velocity  $v$ . In this case, provided the wall is not an idealized perfect conductor, there is a velocity-dependent force  $F(v)$  acting along the direction of motion of the atom. This force can be associated physically with the effect of the finite conductivity of the wall material on the image field of the atom. The functional form of  $F(v)$  depends sensitively on how the dielectric function of the wall material varies with frequency [22]. Pendry [23] has discussed the possibility of a frictional force when two infinite parallel mirrors separated by a fixed distance are in relative motion, and finds “large frictional effects comparable to everyday frictional forces provided that the materials have resistivities of the order of 1 m $\Omega$  and that the surfaces are in close proximity.” As in the case of an atom moving with respect to a wall, the form of the frictional force depends sensitively on the dielectric function. In fact, a *Gedanken* experiment suggests that lateral Casimir forces are present even for ideal finite conducting parallel planes, otherwise it would be possible to construct a device that would extract a net positive energy from the vacuum in each cycle of its operation [24].

### 3 Technological Implications

We have already alluded to what may be some profoundly important aspects of the quantum vacuum, and have noted that the reality of various Casimir effects lends credibility to predictions of various vacuum field effects that lie fantastically beyond the pale of experiment. Of course Casimir effects are also of interest in their own right and, if anything, this interest appears to be growing. Moreover, recent work – including that on vacuum friction – suggests that Casimir effects may be of some practical significance.

Casimir effects will be significant in microelectromechanical systems (MEMS) if (when) further miniaturization is realized [25]. Smaller distances between MEMS components are desirable in electrostatic actuation schemes because they permit smaller voltages to be used to generate larger forces and torques. MEMS currently employed in sensor and actuator technology have component separations on the order of microns, where Casimir effects are negligible.

However, the Casimir force per unit area between perfectly conducting plates,  $F = -\pi^2 \hbar c / 240 d^4$ , increases rapidly as the separation  $d$  is decreased; at a separation of 10 nm,  $F \sim 1$  atm.

Serry *et al* [25] have considered an idealized MEMS component resembling the original Casimir example of two parallel plates, except that one of the plates is connected to a stationary surface by a linear restoring force and can move along the direction normal to the plate surfaces. The Casimir force between the two plates, together with the restoring force acting on the moveable plate, results in an “anharmonic Casimir oscillator” exhibiting bistable behavior as a function of the plate separation. This suggests the possibility of a switching mechanism, based on the Casimir effect, that might be used in the design of sensors and deflection detectors [25]. The analysis also suggests that the Casimir effect might be responsible in part for the “stiction” phenomenon in which micromachined membranes are found to latch onto nearby surfaces.

An experimental demonstration of the Casimir effect in a nanometer-scale MEMS system has recently been reported [26]. In the experiment the Casimir attraction between a 500  $\mu\text{m}$ -square plate suspended by torsional rods and a gold-coated sphere of radius 100  $\mu\text{m}$  was observed as a sharp increase in the tilt angle of the plate as the sphere-plate separation is reduced from 300 nm to 75.7 nm. This “quantum mechanical actuation” of the plate suggests “new possibilities for novel actuation schemes in MEMS based on the Casimir force” [26].

## 4 Complications and Approximations

Calculations of Casimir forces for situations more complicated than two parallel plates are notoriously difficult, and one has little intuition even as to whether the force should be attractive or repulsive for any given geometry. The fact that the Casimir force on a perfectly conducting spherical shell is *repulsive*, as first discovered by Boyer [27], surprised even Casimir, who presumed that the force would be attractive [28]. Since Boyer’s work a fairly large literature has grown around problems of calculating Casimir forces for perfectly conducting spheres, cubes, cylinders, wedges, and other geometries.

In the case of dielectrics the situation is even more complicated, as can be appreciated from the Lifshitz theory for the “simple” example of two parallel walls (see, for instance, Milonni 1994, pp 219-33). Computation of the numerical value of the force per unit area requires a knowledge of the complex refractive index as a function of frequency as well as the deviation from perfect surface smoothness; therefore, as discussed by Lamoreaux [29] and Klimchitskaya *et al* [30], for instance, accurate computations require auxiliary measurements of various properties of the surfaces.

It would be very useful to have approximate methods for the calculation of Casimir forces for arbitrarily shaped bodies. The obvious and simplest approximation is to add up *pairwise* van der Waals forces [31]. Consider, for instance, an atom  $A$  at a distance  $d$  from a half-space of  $N$  atoms per unit volume, and

suppose that all the atoms are identical and that  $d$  is large enough that the interaction between  $A$  and each atom of the “wall” is the retarded van der Waals interaction  $V(r) = -23\hbar c\alpha^2/4\pi r^7$ . Adding the pairwise interactions between  $A$  and all the wall atoms, one easily finds

$$V(d) = -\frac{23}{40} \frac{\hbar c\alpha}{d^4} N\alpha . \quad (2)$$

Now if we use the Clausius-Mossotti relation between  $N\alpha$  and the dielectric constant  $\epsilon$ , and assume that the limit  $\epsilon \rightarrow \infty$  should correspond to a perfectly conducting wall, then the potential energy of  $A$  when it is at a distance  $d$  from a perfectly conducting plate should be

$$V(d) = -\frac{69}{160\pi} \frac{\alpha\hbar c}{d^4} \quad (3)$$

in the pairwise approximation. This is 15% larger than the Casimir-Polder result cited earlier. A similar calculation of the pairwise van der Waals force per unit area between two parallel walls gives

$$F(d) = -\frac{207\hbar c}{640\pi^2 d^4} , \quad (4)$$

which is 20% smaller than the Casimir result [2].

In light of the stark simplicity of the pairwise approach, these results are certainly encouraging. However, these two examples pretty much exhaust the supply of known, exact results for the Casimir interaction of disconnected objects. Let us consider therefore some known results for connected objects.

We have already alluded to the Casimir energy for a perfectly conducting spherical shell. A calculation of the pairwise van der Waals energy of a spherical ball of radius  $a$  gives  $V(a) = (207\hbar c/1536\pi a)[(\epsilon - 1)/(\epsilon + 2)]^2$  if we ignore dispersion and assume again that  $N\alpha$  and  $\epsilon$  are related by the Clausius-Mossotti formula. Thus  $\lim_{\epsilon \rightarrow \infty} V(a) = .043\hbar c/a$  for a spherical ball is of the same order as the exact result  $(.09\hbar c/a)$  for the conducting spherical shell [27] and gives the correct, “counter-intuitive” sign. (Note: We disregard infinities associated with the divergence of the van der Waals interaction when the intermolecular spacing goes to zero, as assumed in our continuum model. That is, we retain only what Barton [32] refers to as the “pure Casimir term.”)

Unfortunately the surprising degree of accuracy of this (relatively) simple approach in these examples seems fortuitous. For the case of an infinitely long conducting cylindrical shell of radius  $a$ , the pairwise approach gives a Casimir energy of zero, whereas several authors have found that there is an attractive force. (See Barton 2000, Reference [32], and references therein.) In the case of a conducting cube of side  $a$  the exact calculation yields  $V(a) = .092/a$ , whereas Barton finds that the pairwise approximation gives an *attractive* Casimir term [32]. (After initially obtaining a repulsive force, and checking the laborious calculations after learning of Barton’s result, we have confirmed that the “pure Casimir term” is indeed attractive in the pairwise approximation.)

Ambjorn and Wolfram [33] remarked that “[the pairwise approximation] ... in the case of two parallel planes ... leads to a correct Casimir energy,” but that “this result is probably fortuitous.” They support this claim by remarking that, “according to [the pairwise approximation] the Casimir forces between conducting surfaces would ... always be attractive,” whereas for the cube, for instance, the actual Casimir force is repulsive, as they showed. We note, however, that the pairwise approximation for the sphere gives in fact a repulsive force for the pure Casimir term.

It appears then that there is still no reliable approximation to the evaluation of Casimir forces for arbitrarily shaped bodies. It is worth noting, however, that Schaden and Spruch [34] have developed a semiclassical approach that might lend itself to workable approximations for arbitrary geometries.

## 5 Acknowledgement

In this tribute to Casimir we have tried to convey our strong belief that, after all these years, there is still much to appreciate and learn about Casimir effects. We are grateful not only to Casimir but to our many colleagues whose work has kept the subject alive and well. For the discussion in the final section we are particularly grateful to Gabriel Barton for sharing his results with us prior to their publication. GJM would like to thank the NASA Breakthrough Propulsion Physics program for its support of this work.

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