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Vacuum Forces in Microcavities

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Abstract

According to calculations in quantum electrodynamics, rectangular microcavities in metals should display repulsive as well as attractive forces on the walls, depending on the ratio of the sides. We consider some dynamical structures using cavities in which vacuum forces are important and an experiment designed to detect the existence of repulsive forces between an array of rectangular cavities and a metallized sphere mounted on the tip of an atomic force microscope.

Introduction

Casimir forces arise from the change in the ground state of the electromagnetic field due to the presence of surfaces. The vacuum electromagnetic field must meet the appropriate boundary conditions at the surfaces. Therefore the ground state field is different from the ground state field when no matter is present. From a knowledge of the fields fields one can compute the stress-energy tensor and determine the forces on the surfaces[1][2][8]. A force in a given direction can obtained directly from knowledge of the spatial part of the stress-energy tensor or a as a derivative of the vacuum energy with respect to the corresponding coordinate. In order to obtain a finite answer, one generally computes the change in the vacuum energy due to the presence of the surfaces[3]. Most effort has gone into understanding the forces and related physics for two infinite parallel conducting plates. In fact, the parallel plate geometry is the **only** geometry that we are aware of for which there is a rigorous calculation of the force between two separate macroscopic bodies[4]. Corrections for surface roughness, finite conductivity, and temperature have also been computed to establish agreement between theory and experiment at the 1% level[5][6][7].

The parallel plate configuration has several pleasing features from the theoretical point of view: 1. It preserves Lorentz invariance in two dimensions; 2. It reduces the problem effectively to a one dimensional problem; 3. The stress-energy tensor is constant in the region between the plates; 4. There is no curvature of the surfaces[9]. For the infinite parallel plate geometry, the vacuum energy $E_{pp}(a)$ is negative and the vacuum force/area $F_{pp}(a)$ is attractive

$$E_{pp}(a) = -\frac{K}{3a^3} \tag{1}$$

$$F_{pp}(a) = -\frac{K}{a^4} \tag{2}$$

where a is the separation between the plates, $K = \pi^2 \hbar c/240$.

Vacuum energy calculations have been done for a cube and a spherical shell formed with conductive walls[10][11]. In both these cases the vacuum energy is found to be positive and the force is outward or repulsive. Because of the very special nature of the parallel plate geometry and the high degree of symmetry of the cube and sphere, it is not reliable to make generalizations about the behavior of vacuum forces based on these special geometries.

The vacuum forces on the faces of conductive rectangular boxes or cavities show very different features compared to those of the parallel plate, the cube, and the sphere[12][13][14]. For a rectangular cavity, the total force on a given face can be positive, zero, or negative depending on the ratio of the sides of the box. In fact there are cavities that have zero force on two sides and a positive or negative force on the remaining side. The are boxes for which the energy is negative (or positive) and the forces on some walls are attractive while the forces on the remaining walls are repulsive. Indeed it is difficult to get an intuitive picture of the meaning of these results.

It is of interest to determine what physical consequences might result from this complex behavior of vacuum forces in rectangular cavities, and to explore the possibility of measuring repulsive vacuum forces in a cavity geometry experimentally.

I. VACUUM ENERGY AND FORCES IN CONDUCTING RECTANGULAR CAVITIES

The vacuum energy of a perfectly conducting rectangular parallelepiped cavity has been computed using several methods that all yield the same value for the finite part of the energy shift[12]. There are divergences due to the perfect right angles of the edges and corners that do not alter our conclusions[15]. The vacuum energy $en(a_1, a_2, a_3)$ for a rectangular cavity with sides (a_1, a_2, a_3) is given in terms of a generalized Epstein zeta function:

$$en(a_1, a_2, a_3) = \frac{-\hbar c}{16\pi^2}[a_1 a_2 a_3 \sum_{n_3=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_1=1}^{\infty}\left[(a_1 n_1)^2 + (a_2 n_2)^2 + (a_3 n_3)^2\right]^{-2} - \frac{\pi^3}{3}(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3})]. \tag{3}$$

The summation always gives a negative contribution to the energy, while the second term in brackets gives a positive contribution. Since the vacuum energy is proportional to $\hbar c/length$ and the energy of the cavity is independent of the labeling of the sides, the energy $en(a_1, a_2, a_3)$ is a homogeneous function of $a_1, a_2, a_3 : of$ degree -1

$$e(\lambda a_1, \lambda a_2, \lambda a_3) = \lambda^{-1} en(a_1, a_2, a_3). \tag{4}$$

Surfaces of constant energy are defined by the relationship $en(a_1, a_2, a_3) = K$, where K is a constant. Because the energy is a homogeneous function, all positive energy surfaces are identical to each other with a scale change; all negative energy surfaces are identical with a scale change. The forces on the walls due to the ground state of the electromagnetic field

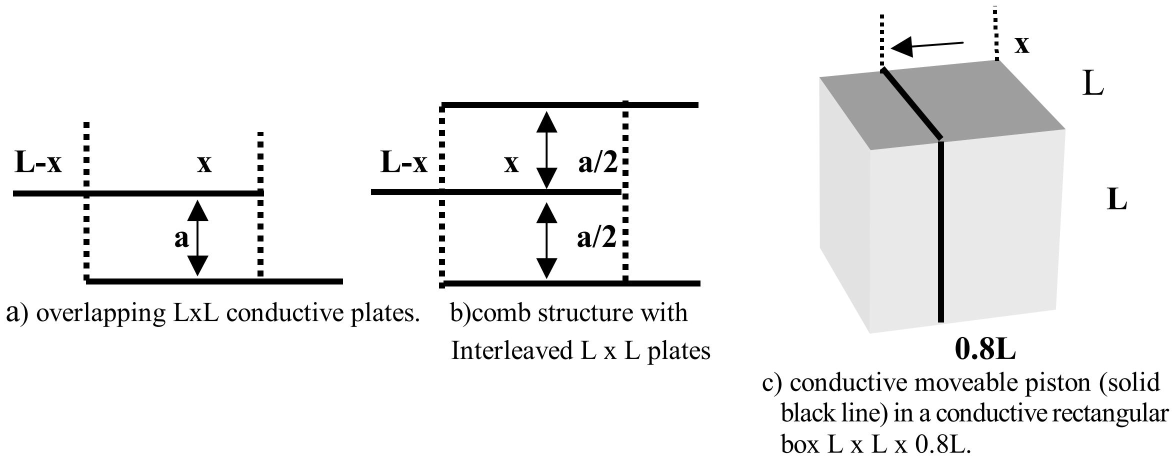

FIG. 1: Several different structures that involve vacuum forces.

can be obtained as the negative of the gradient of the vacuum energy:

$$\mathbf{F}(\mathbf{a}) = -\boldsymbol{\nabla}_a en(\mathbf{a}) \tag{5}$$

where the component $F_1 = -\partial en(\mathbf{a})/\partial a_1$ is the total force on the face normal to the a_1 direction, etc. The force $\mathbf{F}(\mathbf{a})$ is normal to the corresponding surface of constant energy at $\mathbf{a}$. From Eulers theorem for homogeneous functions, we have

$$\mathbf{a} \cdot \mathbf{F}(\mathbf{a}) = e(\mathbf{a}). \tag{6}$$

II. SIMPLE DYNAMICAL STRUCTURES

We can make some approximate calculations to explore the possible motion of surfaces in cavity geometries[16]. For these calculations we assume perfectly conducting walls, zero temperature, and neglect effects of any divergences or right angles. We consider the forces and energy balance in several simple structures (as shown in Fig. 1) with: 1. Lateral force for overlapping parallel plates; 2. Casimir force comb drive; 3. Frictionless piston moving in a rectangular cavity.

A. Lateral Casimir force for finite parallel plates

Although no one has yet computed in detail the lateral forces between finite, offset, parallel, conducting plates, it is very probable that such forces are not zero, otherwise one could extract energy from the vacuum during each cycle of operation of a machine in which only the Casimir forces operated. Since the electromagnetic field is a conservative field, this is not possible which implies that lateral forces must exist. We can use a heuristic model to compute the lateral Casimir force acting on two square LxL parallel plates. If we neglect vacuum energy "fringing fields," assume that $a << L$ and that the energy density differs from the free field density only in the region in which the square (LxL) plates overlap a distance x, where $0 < x < L$ (see Fig. 1a), then we can compute the lateral force F_{L2} between the two plates using the conservation of energy (principal of virtual work):

$$F_{L2}(x) = -\frac{d(-KLx/3a^3)}{dx} = \frac{KL}{3a^3} \tag{7}$$

which yields a constant lateral force that tends to increase x or pull the plates so they have the maximum amount of overlap. The usual Casimir force that is normal to the plates when they are directly opposite each other, with complete overlapping (x = L), is L/3a times larger than the constant lateral force. The approximate formula forF_{L2} is based on equations for the infinite parallel plate energy density which requires that $x >> a$. Thus for small x we would expect deviations from this formula. When one includes the work done by this constant attractive lateral force then there is no net change in total energy (mechanical plus field) as x goes from 0 to L. When we include this force it is not possible to construct machines that violate energy conservation in a conservative field. Lateral forces for infinite plates also are predicted for finite conductivity plates[17].

B. Casimir force comb drive

Consider the case of two fixed, square (LxL), parallel plates separated by a distance a, with a third moveable plate that slides in between the two parallel plates, separated by a distance $a/2$ from each plate (Fig. 1b). If we neglect vacuum energy "fringing fields", as before, assume that the vacuum energy is different from zero only in regions between directly opposing plates, and that we can apply the infinite parallel plate formula for energy density, then we can compute the lateral force on the moveable plate in the middle as minus the derivative of the vacuum energy $E_{L3,}$where the energy can be written as a function of the overlap x of the fixed and moveable plate

$$E_{L3}(x) = -\frac{2KLx}{3(a/2)^3} - \frac{KL(L-x)}{3a^3}. \tag{8}$$

Differentiating with respect to x yields a constant lateral force equal to

$$F_{L3} = \frac{5KL}{a^3}. \tag{9}$$

This force is 5a/L times the normal Casimir force between the parallel plates separated by a distance a. . For a device with a = 0.1 μm, L = 1 mm, the lateral force would be an easily measurable 31 nanoNewtons. This structure is analogous to the electrostatic comb drive that is used extensively in MEMS (microelectromechanical systems) devices. One key operational difference between the Casimir and electrostatic drives is that the Casimir force drive always yields an inward or attractive force, whereas the voltage on the electrostatic comb drive can be reversed in polarity, reversing the direction of the force. Another difference is that the Casimir force comb drive requires no external power source, whereas the electrostatic drive does.

C. Rectangular cavity with a moveable piston

Consider a rectangular conductive cavity ($LxLxa$) with a frictionless, moveable piston that moves along the a-direction, dividing the cavity into two regions, each with its contribution to the total vacuum energy. We assume the piston is infinitely thin and normal to the a-direction (Fig. 1c). We can then numerically compute the total vacuum energy $E_p(L, L, x)$ of the structure as a function of the distance x of the piston from one end of the cavity. From our definition of $en(a1, a2, a3)$, it follows that

$$E_p(L, L, x) = [en(1, 1, (a - x)/L) + en(1, 1, x)]/L \tag{10}$$

where we have assumed that the total vacuum energy is just the sum of the energy for each cavity. If we differentiate the total energy with respect to x, we obtain an expression for the force $F_p(x)$ due to the vacuum stresses on the moveable plate. Consider an example in which $a = 0.8L$. Figure 2 shows the dimensionless energy and force $LEp(L, L, x)$ and $L^2F(x)$ respectively as functions of $20x$. For values of the abscissa near the center of the cavity ($20x \sim 8$), the force on the piston is approximately directly proportional to x, and the energy is approximately a negative parabola with negative curvature.

For these calculations in Figure 2, we have set $\hbar c = 1$ and set L equal to 1 unit. To obtain a numerical result, we use the MKS value of $\hbar c$. If we let L=0.5 μm, then the abscissa is

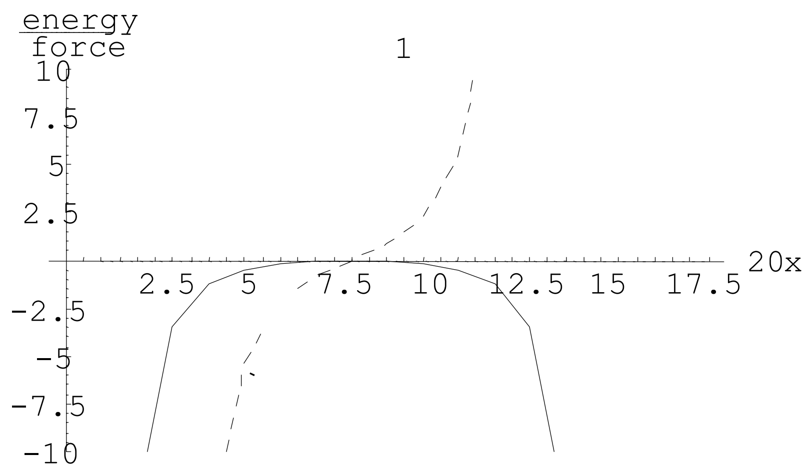

FIG. 2: Plot of the vacuum energy (___solid line) and Casimir force (_ _ _ dashed line) for a 1 x 1 x .8 cavity that is divided into two rectangular cavities by a sliding piston that moves along the 0.8 direction.

in units of 0.025 μm, the energy scale is in units of $6.3x10^{-20}$ joule, and the force scale is in units of $1.26x10^{-13}$ Newton. Forces of this magnitude are just detectable using current AFM technology.

A small deflection from the center leads to a force causing an increased deflection. Thus the equilibrium state of the system with the piston at the center is unstable: after any deflection from the exact center of the cavity, the piston would be pushed by quantum vacuum forces to the closest end of the cavity. This unstable behavior is common to a large variety of cavity shapes.

D. Vibrating cavity wall

The unstable behavior mentioned above also appears in a single cavity of proper dimensions. For example, a cavity that has sides in the approximate ratios 0.15 x 0.10 x 2 will have zero force on the face normal to the 0.15 direction. The zero force corresponds to an unstable energy maximum. Thus a deflection inward leads to an increasing inward (attractive) force, and, conversely, any deflection outward (repulsive force) leads to an increasing outward force. This is akin to a harmonic oscillator, except the force is destabilizing (F=kx) rather than stabilizing (F=-kx). If we assume that the box is made of real conductive materials, then there will be a restoring force due to the material. If we include the restoring force that arises from the small deflection of a deformable membrane as given by Hooke's Law, then this configuration might become stable if the material force constant exceeds that for the Casimir force. These results suggest the intriguing possibility of making a structure that displays simple harmonic motion for small displacements. Thus the face of a box of the proper dimensions may oscillate under the mutual influence of the vacuum force and the Young's modulus of the material. The oscillations would be damped due to the non-ideal properties of the material and the friction with the environment. A zero point oscillation of the cavity wall would be expected. The energy in the lowest mode would be modified by the temperature.

III. MEASUREMENT OF REPULSIVE FORCES

To date the only vacuum force that has been measured for conducting surfaces is the force between an approximately flat surface and a slightly curved surface[5][6]. The "flat" surfaces are characterized by a finite surface roughness, which effectively increases the mean separation and reduces the vacuum force. The curved surface is spherical and is used to avoid the experimental difficulties in obtaining two flat surfaces that are parallel to microradians. In different experiments, the spherical surface has been a metallized lens or a metallized sphere mounted on an AFM cantilever. One recent experiment was also done with a flat surface which had 50 nm high corrugations with a period of a μm and a sphere on an AFM in order to show that the vacuum force depends on the boundary conditions in a way that cannot be predicted by a simple perturbation theory model[18].

The QED calculations discussed earlier show that repulsive forces are predicted to be present in a variety of conductive rectangular cavities[12]. The question is: If we cut such a cavity into two pieces, with a cut being in a plane parallel to one of the faces, will we find a repulsive force between the two separate surfaces? Or will an attractive force between the edges dominate? No computations have yet been done for this situation[19]. In order to determine if repulsive forces between separate surfaces are possible, we are performing an experiment in which a metallized sphere on an AFM is brought to within 10's of nanometers of an array of cavities in gold[20]. If repulsive forces are present we should observe a corresponding deflection in the sphere. A schematic of the experiment is shown in Figure 3.

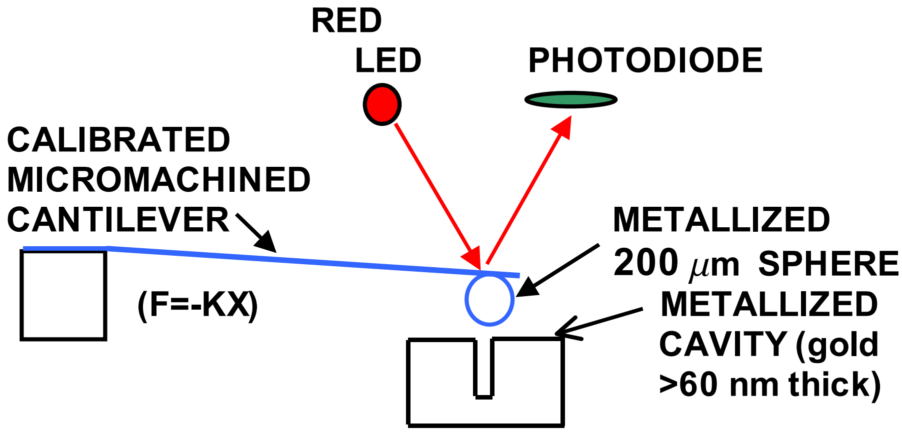

FIG. 3: Schematic of experiment to measure repulsive vacuum forces using sphere mounted on an AFM.

The force from one cavity in not large enough to give a measurable deflection to the AFM. Hence we actually use an array of cavities. Our target array is composed of 500 cavities in a 100 μm x 100 μm array. Each cavity in this array would have a cavity width w of 0.1 μm, with wall thickness t of 0.1 μm. Cavities are 100 μm long and 0.6 μm deep. Since these dimensions are so small, and the aspect ratio is large, they are being made using x-ray lithography at the University of Wisconsin Center for NanoTechnology. To date our best cavities have a width of 125 nm, a wall thickness of 245 nm, and a depth of 0.4 um. The geometry of the spherical AFM tip and the cavity array is shown in Figure 4.

There are no theoretical predictions about the force between a spherical surface and a cavity. Hence we have developed a heuristic model, that doubtless will be modified when the correct theory is developed. We assume that the total force between the sphere and the cavity array is composed of two terms, an repulsive force due the cavity itself, and an attractive parallel plate force due to the attraction between the tops of the cavity walls and the sphere. For the attractive wall force, we multiplied the ideal, infinite, parallel plate attractive force by a correction factor to account for the fact that we do not have an infinite parallel plate geometry:

$$\frac{t}{t+\delta}. \tag{11}$$

For the repulsive cavity force we used the numerically computed force on a closed cavity

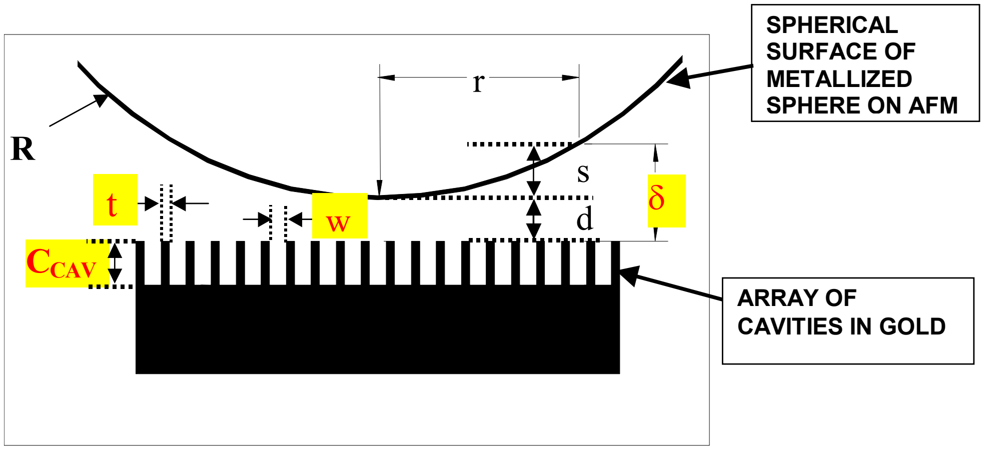

FIG. 4: Schematic showing the surface of the sphere mounted on an AFM cantilever and the cavity array for an experiment to measure repulsive Casimir forces.

with a depth equal to the actual depth ($Ccav$) and multiplied it by the correction factor

$$\left(\frac{C_{cav}}{C_{cav}+\delta}\right)^3\left(\frac{w}{w+\delta}\right). \tag{12}$$

which is suggested by theoretical analysis of the stress-energy tensor near curved surfaces[21]. The term linear in δ is included to account for fringing effects. Summing the attractive and repulsive forces, we obtain an expression for the net pressure as a function of the distance above the cavity array (Figure 5a) When this force is integrated over the surface of the sphere, we obtain curves of force versus separation (Figure 5b). For our target geometry (w=t=0.1 μm), there is a clear transition in Fig 5b in which the force changes sign, and a measurably long region in which the force is repulsive. For our smallest cavities to date, which are significantly larger(t=0.245μm, w=0.125 μm) than the target geometry, the theory predicts neither a clearly measurable transition from negative to positive forces nor a clearly measurable positive force for any separation.

If experiment verifies that repulsive forces, as wall as attractive forces, can be present between two separate bodies, it may allow us to design a variety of novel MEMS devices.

IV. CONCLUSION

The magnitude and the sign of the Casimir force depends strongly on the specific geometry. Although the most extensively studied geometry investigated to date, namely the

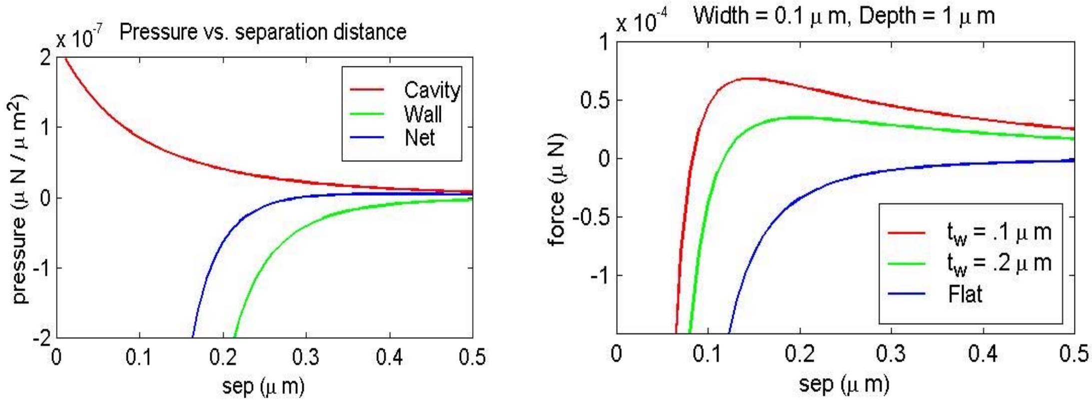

FIG. 5: (a) The attractive, repulsive and net pressure for cavity width w = 0.2 μm, wall thickness t = 0.1μm , 0.5um depth.(b) Total force on the AFM sphere for the target array (t = w = 0.1 μm), and an array with wall thickness t = 0.2 μm, and a flat surface.

parallel plate geometry, only displays attractive forces, other geometries display attractive and repulsive forces as well. Rectangular conductive boxes exhibit a broad variety of behavior, with the sign and magnitude of the vacuum force and vacuum energy, depending on the ratio of the sides of the cavity. It is possible to construct model devices using these cavities. An experiment is proposed to measure the repulsive force between a metallized sphere on an AFM tip and an array of rectangular cavities etched in gold.

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